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Surveying

BOOK III

By

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STADIA AND PLANE-TABLE SURVEYING
TOPOGRAPHIC SURVEYING
HYDROGRAPHIC SURVEYING

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STADIA AND PLANE-TABLE SURVEYING

STADIA AND PLANE-TABLE SURVEYING
(*Continued*)

TOPOGRAPHIC SURVEYING

HYDROGRAPHIC SURVEYING

STADIA AND PLANE-TABLE SURVEYING

Serial 5221 Edition 1

STADIA SURVEYING

DETERMINATION OF DISTANCES BY STADIA

PRINCIPLE OF STADIA MEASUREMENTS

1. Essential Features of Stadia Method.—The stadia method of surveying is advantageously employed in various kinds of surveys, because it furnishes a quick and convenient way of determining moderate or comparatively long horizontal distances where a high degree of accuracy is not required. This method is also used advantageously for determining differences in elevation between points. In determining either the horizontal or the vertical distance between two points by the stadia method, the procedure is essentially as follows:

A transit equipped with two special horizontal cross-hairs is set over one point and is sighted at a graduated rod held on the other point. The vertical distance intercepted on the rod between the two cross-hairs is observed, and the required horizontal or vertical distance between the two points is computed from the observed intercepted distance and other known values.

2. Applications of Stadia Method.—The more important applications of the stadia method of surveying may be outlined as follows:

1. To determine the approximate lengths of foresights and backsights in direct leveling, in order to balance these lengths.
2. To determine the horizontal distances from the instrument to various points on which the rod is held in direct level-

ing. Thus, both the elevations of the points and the distances to them are established.

3. To determine the lengths of the courses of a traverse, either for the purpose of checking chained measurements or for making original measurements of low precision.

4. To determine the positions and elevations of points, especially in topographic and hydrographic surveys, by sights from previously established points.

5. To determine differences in elevation by indirect or trigonometric leveling.

3. Equipment for Stadia Work.—The telescope of a transit that is to be used for stadia work must be equipped with two special horizontal cross-hairs, which are called *stadia hairs*, or *stadia wires*, and are set a certain distance apart. The distance between the stadia hairs is known as the vertical interval between the stadia hairs. These hairs are independent of the usual horizontal cross-hair, which is midway between the two stadia hairs and is generally called the *middle hair*, or *middle wire*.

In some telescopes, focusing of the objective lens is accomplished by moving that lens toward or away from the eyepiece. However, many instruments for stadia work are provided with internal focusing telescopes. In such telescopes, the objective lens is immovable and there is an internal focusing lens that can be moved forward or backward so as to make distinct any image formed by the objective.

For comparatively short sights, an ordinary leveling rod may be used for stadia work, but for longer sights a special rod that is wider and has larger graduations is needed. Such a rod is called a *stadia rod*.

4. Properties of Lenses.—Measurement by stadia is based on certain fundamental principles of lenses. The manner in which rays of light are deflected by a lens is illustrated diagrammatically in Fig. 1. In view (*a*) the rays of light are assumed to come to the lens, which is shown cross-hatched, from such a great distance that they may be considered parallel to each other. They are then deflected by the lens so as to pass through

a certain point F, called the *principal focus* of the lens. The distance from the center of a lens to its principal focus is known as the *focal length of the lens*.

In view (b) the rays of light come from some point A that is at a moderate distance from the lens and are deflected by the lens so as to pass through a certain point A', which is beyond

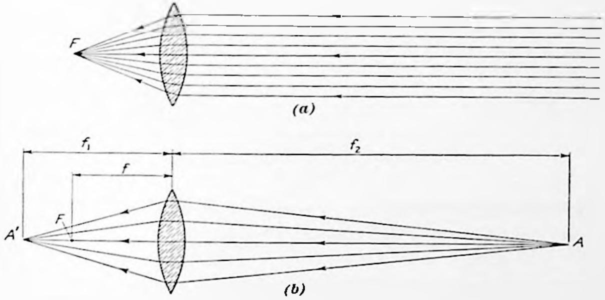


FIG. 1

the principal focus F. Likewise, rays of light from A' would be deflected so as to pass through A. Any two points, such as A and A', which are so located that rays of light from one will pass through the other are called *conjugate foci* (foci is the plural of focus).

It is a law of lenses that the sum of the reciprocals of the distances from a lens to two conjugate foci is equal to the reciprocal of the focal length of the lens. In other words,

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f}$$

in which f_1 and f_2 are the distances from the lens to conjugate foci, as indicated in view (b), and f is the focal length.

5. Basis of Stadia Method.—In Fig. 2 are represented diagrammatically the conditions for a stadia measurement that is made when the line of sight is horizontal and a telescope without internal focusing is used. When an internal focusing telescope is used, the conditions are essentially similar. It is

assumed that the actual stadia hairs are located in a vertical plane at a and b, and that the apparent positions of these hairs on a vertical rod are at A and B, respectively. A ray of light may pass from a to A, or from b to B, along a line through the center of the objective lens, which is shown cross-hatched. Actually, such a ray is deflected at each surface of the lens; but, for the purpose of this explanation, the rays may be considered straight, as shown in the illustration. Also, a ray of light from a may reach A by following the path $aa'A$, and a ray

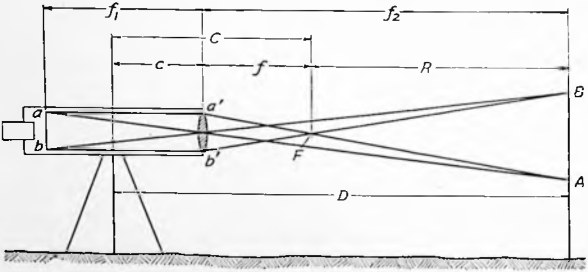


FIG. 2

from b may take the path $bb'B$. According to the theory of lenses, the lines aa' and bb' are parallel to the axis of the telescope, which in this case is horizontal, and the lines $a'A$ and $b'B$ pass through the principal focus F of the lens. Moreover, the planes ab and AB will pass through conjugate foci of the lens.

In the case of a surveying instrument, the distance f_2 from the objective lens to the rod is always very large in comparison with the distance f_1 from the lens to the stadia hairs. Therefore, the value of the fraction $\frac{1}{f_2}$ is very small in comparison with the value of $\frac{1}{f_1}$ and, for the purposes of stadia work, it may be assumed that

$$\frac{1}{f_1}=\frac{1}{f} \text{ or } f_1=f$$

6. An imaginary vertical line joining the points a' and b' in Fig. 2 would be parallel to the line AB. Then, since the lines $a'A$ and $b'B$ are straight, the triangle $a'b'F$ would be similar to the triangle ABF and corresponding dimensions of these triangles would be proportional. Hence, if R denotes the distance from the principal focus of the lens to the rod at AB,

$$R:f = AB:a'b'$$

But the lines aa' and bb' are parallel, and $a'b' = ab$. If the vertical interval ab between the stadia hairs is designated as i, and the interval AB intercepted on the rod between the stadia hairs is designated by s, it follows from the preceding relation that

$$R=\frac{f}{i}\times s$$

The distance that is required in practical work is the distance D, Fig. 2, from the center of the instrument to the point on which the rod is held. As shown in the illustration, the distance D is equal to the sum of the distance R from the rod to the principal focus of the objective, the focal length f from that focus to the lens, and the distance c from the lens to the center of the instrument. Thus,

$$D=R+(f+c)=\frac{f}{i}\times s+(f+c)$$

INSTRUMENT CONSTANTS

7. **Stadia Constant.**—The distance c, Fig. 2, from the center of a transit to the objective varies slightly as the objective is focused for sights of different lengths. However, this variation is so small in comparison with the distance from the instrument to the rod that it may be neglected and the distance c may be considered constant. Since the focal length f is obviously fixed for a particular instrument, it is customary to treat the sum of f and c as a constant. The total distance $f + c$ is called the stadia constant.

In the case of a telescope without internal focusing, the value of the stadia constant is usually determined by the manufacturer and is marked on a card that is attached to the inside of the instrument box. This value ranges between 0.6 foot and

1.4 feet. In practice, it is frequently taken as 1 foot. The error due to this approximation is usually unimportant, and the computations are simplified. When the instrument is equipped with an internal focusing telescope, the stadia constant is eliminated from the computations for both horizontal and vertical distances.

If it is desired to determine by observation the stadia constant for an instrument without internal focusing, the following method is sufficiently accurate. The focal distance f, which is assumed to be equal to the distance f_1 in Fig. 1, is found by focusing the objective on a distant object and then measuring the distance from the objective to the cross-hair ring. Also, the distance c is found by focusing the objective for an average length of sight and measuring the distance from the objective to the center of the instrument.

8. Stadia Hairs and Stadia Interval Factor.—Stadia hairs are mounted in telescopes in various ways. Usually, the stadia hairs are set by the instrument manufacturer in fixed positions on the same ring and in the same plane with the regular horizontal and vertical cross-hairs. Most instrument makers then attempt to place the stadia hairs in telescopes without internal focusing so that the ratio $\frac{f}{i}$, or the ratio of the focal length of the objective to the actual distance between the stadia hairs, will be exactly equal to 100. If the instrument has an internal focusing telescope, the maker sets the stadia hairs so that correct results are obtained when the ratio $\frac{f}{i}$ is taken as 100 and the instrument constant is neglected. In any case the stadia hairs are intended to be parallel to and equidistant from the middle hair.

When the stadia hairs and the regular hairs are mounted in the same plane, all the hairs are visible at the same time, and care must be exercised not to confuse a stadia hair and the middle hair. In order to avoid the possibility of such confusion, the stadia hairs and the regular hairs are sometimes mounted in different planes. The stadia hairs are then in-

visible when the regular hairs are properly focused, and vice versa. Such stadia hairs are called *disappearing stadia hairs.*

The instrument maker can usually set the stadia hairs in the desired positions with sufficient accuracy. However, in case the ratio $\frac{f}{i}$ is not made exactly 100, the computations of the distances from stadia readings are rendered somewhat more cumbersome. Instruments were formerly provided with adjustable stadia hairs, which could be set so as to obtain any desired value of the ratio $\frac{f}{i}$. Since the positions of fixed hairs cannot be changed accidentally, whereas the setting of adjustable hairs may be disturbed by accident, fixed stadia hairs are generally preferred.

For any telescope with fixed stadia hairs, whether or not they are disappearing, the value of the ratio $\frac{f}{i}$ is a constant. This ratio is known as the stadia interval factor.

9. Determination of Stadia Interval Factor.—The value of the stadia interval factor of a particular instrument may be readily determined or verified. The field work is as follows: First, a straight line having a length of 400 to 800 feet is laid out on ground that is practically level. The rod is then held successively at various points on this line at measured horizontal distances from the instrument and, with the line of sight horizontal in every case, the space intercepted between the stadia hairs on the rod at each point is observed and recorded.

In order to compute the desired ratio $\frac{f}{i}$, the formula $D = \frac{f}{i} \times s + (f+c)$ is applied in the following manner: If the known horizontal distances from the center of the instrument to any two points are represented by D_1 and D_2, respectively, and the respective rod intervals are denoted by s_1 and s_2, then

$$D_1 = \frac{f}{i} \times s_1 + (f+c)$$

and

$$D_2 = \frac{f}{i} \times s_2 + (f+c)$$

When the second of these equations is subtracted from the first, the result is

$$D_1 - D_2 = \frac{f}{i} \times s_1 - \frac{f}{i} \times s_2 = \frac{f}{i} \times (s_1 - s_2)$$

and

$$\frac{f}{i} = \frac{D_1 - D_2}{s_1 - s_2}$$

In other words, whether or not the instrument has an internal focusing telescope, the stadia interval factor can be computed by taking the difference between the horizontal distances from the instrument to two points and also taking the difference between the rod intervals for the same two points, and then dividing the first difference by the second difference. The accuracy of the adopted value may be increased by applying the method to several pairs of points, and taking the average of the results thus obtained.

In a test of this kind, the horizontal distances should be carefully measured with a steel tape, and the rod interval should be read as closely as possible by means of a target and vernier. If very accurate determinations are desired, the observations should be made when the effect of boiling air or heat waves on refraction will be of least consequence and when the instrument will not be vibrated by wind, as between 8 and 10 o'clock in the morning on a calm, fairly cool day. Also, complete sets of observations should be taken at several different hours of the day.

Example.—Determine the stadia interval factor for a certain transit from the following data:

Distance From Transit to Rod, in Feet	Interval Intercepted on Rod Between Stadia Hairs, in Feet
50	0.488
100	0.989
200	1.989
300	2.990
400	3.992

Solution.—If the observation for the 50-ft. distance is used with each of the other observations in turn, four values of the stadia interval factor will be obtained, as follows:

$$\frac{100 - 50}{0.989 - 0.488} = 99.800$$

$$\frac{200-50}{1.989-0.488}=99.933$$

$$\frac{300-50}{2.990-0.488}=99.920$$

$$\frac{400-50}{3.992-0.488}=99.886$$

The average of these results is 99.885. Ans.

EXAMPLE FOR PRACTICE

A rod is held vertically at several points at measured distances from a transit equipped with stadia hairs, and the intervals intercepted on the rod between the stadia hairs are observed to be as follows:

Distance From Transit to Rod, in Feet	Interval Intercepted on Rod Between Stadia Hairs, in Feet
100	1.000
150	1.504
200	2.011
250	2.516
300	3.019

Compute the value of the stadia interval factor by pairing the first observation with each of the others in turn, and then taking the average of the results thus obtained. Ans. 99.030

FORMULAS FOR STADIA DISTANCES

10. General Conditions.—The formula at the end of Art. 6 may be applied for finding the horizontal distance from the instrument to the rod when the line of sight is horizontal, as indicated in Fig. 2. However, in practical work, the line of sight is usually inclined, and the stadia method may then be adopted for the purpose of determining both horizontal distances and differences in elevation between points.

In Fig. 3 are represented the conditions for an inclined line of sight. In this case, the line of sight is shown elevated above the horizontal, and the vertical angle a of the line of sight is therefore an *angle of elevation*, or a *plus angle*. Similar reasoning applies where the line of sight is depressed below the horizontal, and the vertical angle of the line of sight is therefore an *angle of depression*, or a *minus angle*.

The calculations would be simplified if the rod were held perpendicular to the line of sight. However, it is not practical

to attempt to hold the rod exactly in such a position and, for convenience in the field, the rod is held vertically.

11. Formulas for Horizontal Distance.—In Fig. 3, the line AB represents the vertical rod held on a point O that is to be located by stadia measurement from the instrument point M; and ED represents the inclined line of sight. Also, F is the principal focus of the objective of the instrument, and A and B are the points at which the stadia hairs intercept the vertical rod. If the line $A'B'$ is drawn perpendicular to the line of sight ED, then the inclined distance ED is as given by the equation

$$ED = \frac{f}{i} \times A'B' + C$$

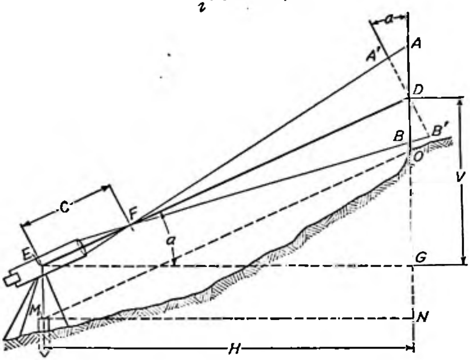


Fig. 3

in which C is the stadia constant $f+c$. This expression for ED is similar to that for the horizontal distance D, Fig. 2, which is derived in Art. 6. Also, in Fig. 3, the angle ADA' or BDB' is equal to the angle a that the line of sight ED makes with the horizontal.

As the angle AFB is very small in any case, no appreciable error in the stadia distance will result if the angles $AA'D$ and $BB'D$ are considered to be right angles. Hence, it may be assumed that $A'B' = AB \cos a$, and

$$ED = \frac{f}{i} \times AB \cos a + C$$

If the product of the stadia interval factor and the distance intercepted on the rod between the stadia hairs is denoted by R, or $R = \frac{f}{i} \times AB$, it follows that

$$ED = R \cos a + C$$

The horizontal distance MN between the points M and O, which is equal to the horizontal distance EG, may be calculated as follows: In the right triangle DEG, $EG = ED \cos a$. Therefore,

$$H = R \cos^2 a + C \cos a \qquad (1)$$

or

$$H = R - R \sin^2 a + C \cos a \qquad (2)$$

in which H = horizontal distance between center of instrument and rod, in feet;

R = product of stadia interval factor and vertical distance intercepted on vertical rod between stadia hairs, in feet;

a = angle that line of sight makes with horizontal:

C = stadia constant, in feet.

The same formula is used whether the vertical angle a of the line of sight is an angle of elevation or an angle of depression. Also, the same result is obtained whether formula 1 or formula 2 is used, and the formula that is preferable in a particular case depends on the method employed for carrying out the calculations. For the vertical angles that are ordinarily encountered in practice, the difference between $C \cos a$ and C will be so small that it may be disregarded. Hence, for most stadia work, the following formulas may be used:

$$H = R \cos^2 a + C \qquad \text{(3) Approx.}$$

$$H = R - R \sin^2 a + C \qquad \text{(4) Approx.}$$

If an instrument with an internal focusing telescope is used, the value of C is generally neglected and the stadia interval factor is taken as 100. The formulas for such an instrument are:

$$H = R \cos^2 a \qquad (5)$$

and

$$H = R - R \sin^2 a \qquad (6)$$

EXAMPLE.—A stadia reading is taken with a transit for which the stadia interval factor is 100 and the stadia constant is 1 foot. What is

the horizontal distance from the transit to a vertical rod, if the length intercepted on the rod between the stadia hairs is 7.21 feet and the angle that the line of sight makes with the horizontal is 18°23'?

SOLUTION.—In formula 3, $R=7.21\times100=721$ ft., $a=18°23'$, and $C=1$ ft. Hence, the required horizontal distance is

$$H=R\cos^2 a+C=721\cos^2 18°23'+1=650.3 \text{ ft.} \quad \text{Ans.}$$

If formula 1 were applied in the preceding example, $C\cos a$, or $1\times\cos 18°23'=0.949$, would be substituted for 1 and the total distance would be called 650.2 feet. However, where the rod is read only to the nearest hundredth of a foot, the final result is accurate only to the nearest foot. Hence, whether formula 1 or formula 3 is used, the distance required in the example would be called 650 feet.

12. Formulas for Difference in Elevation Between Ends of Sight.—From the data used in determining the horizontal distance between two points by the stadia method, it is possible to find also the difference in elevation between the center of the telescope and the point at which the middle hair cuts the rod. These two points are on the line of sight, and may be considered the ends of the sight. In Fig. 3, the difference in elevation between the ends of the sight is represented by the distance DG from D to the horizontal line EG through E. Since $DG=ED\sin a$ and $ED=R\cos a+C$, it follows that

$$DG=R\cos a\sin a+C\sin a$$

But $\sin 2a=2\sin a\cos a$ and $\cos a\sin a=\frac{1}{2}\sin 2a$. Hence, the difference in elevation may be computed by the formula

$$V=R\times\tfrac{1}{2}\sin 2a+C\sin a \qquad (1)$$

in which V = difference in elevation between ends of sight, in feet;

R, a, and C have the same meanings as in Art. 11.

For the vertical angles that are usually encountered in stadia work, the difference between $\frac{1}{2}\sin 2a$ and $\sin a$ is so small that it can be neglected. It is often convenient to substitute $\frac{1}{2}\sin 2a$ for $\sin a$ in formula 1 and to use the following formula:

$$V=(R+C)\times\tfrac{1}{2}\sin 2a \qquad (2)$$

Either of these formulas applies whether the angle a is an angle of elevation or an angle of depression. In many surveys in which differences of elevation are determined by stadia, it is sufficiently accurate to neglect the term $C \sin a$ in formula 1, as it is seldom more than a few tenths of a foot, and to use the formula

$$V = R \times \tfrac{1}{2} \sin 2\,a \qquad \textbf{(3) Approx.}$$

EXAMPLE.—In the example of Art. 11, what is the difference in elevation between the ends of the sight?

SOLUTION.—As given in Art. 11, $R = 721$ ft. and $a = 18°23'$. Then, by formula 3, the required difference in elevation is

$$V = R \times \tfrac{1}{2} \sin 2a = 721 \times \tfrac{1}{2} \sin 36°46' = 215.8 \text{ ft. Ans.}$$

If it is considered necessary to apply formula 1, the required value is

$$V = R \times \tfrac{1}{2} \sin 2a + C \sin a = 215.8 + 1 \sin 18°23' = 216.1 \text{ ft. Ans.}$$

By formula 2,

$$V = (R + C) \times \tfrac{1}{2} \sin 2a = (721 + 1) \times \tfrac{1}{2} \sin 36°46' = 216.1 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1. The line of sight, as determined by the middle hair in a transit, is horizontal, and the interval intercepted on a vertical rod between the stadia hairs of the transit is 4.45 feet. If the stadia interval factor is 100.82 and the stadia constant is 0.9 foot, what is the horizontal distance, to the nearest foot, from the center of the transit to the rod?

Ans. 450 ft.

2. A transit has a stadia interval factor of 100 and a stadia constant of 1 foot. With the line of sight making an angle of 25°21' with the horizontal, the length intercepted on a vertical rod between the stadia hairs is 8.67 feet. (*a*) Determine the horizontal distance from the transit to the rod, to the nearest foot. (*b*) What is the difference in elevation between the ends of the sight, to the nearest tenth of a foot?

Ans. $\begin{cases} (a) \text{ 709 ft.} \\ (b) \text{ 335.9 ft.} \end{cases}$

STADIA REDUCTION TABLE

13. Description of Table.—In order to apply the formulas of Arts. 11 and 12 in the usual manner, it is necessary to perform comparatively long and tedious computations. Where many points are to be located, considerable time can be saved by using tables or diagrams that are based on such formulas or by using a special slide rule. At the end of this text is given a table for reducing stadia readings to horizontal distances and

differences in elevation. In this table are listed the values of 100 $\cos^2 a$ in the columns headed Hor. Dist. and the values of $100\times\frac{1}{2}\sin 2a$ in the columns headed Diff. Elev., for angles ranging from 0° to 30° and varying by intervals of 2′. Thus, for a vertical angle of 6°14′, the value of 100 $\cos^2 a$ is found in the column headed Hor. Dist. under the angle 6° and in the same horizontal line with the number 14 in the column headed Minutes. This value is 98.82, or 100 $\cos^2$ 6°14′=98.82. If the observed vertical angle contains an odd number of minutes, the required values of 100 $\cos^2 a$ and $100\times\frac{1}{2}\sin 2a$ may be readily found by taking the average of the corresponding values for angles 1 minute less and 1 minute greater than the observed angle.

In the last three lines on each page of the table are listed the values of $C\cos a$ and $C\sin a$ for three different values of C. The tabulated values of $C\cos a$ and $C\sin a$ in any column may be used for all vertical angles for which the number of degrees is as given at the head of the column. The correct value of $C\cos a$ or $C\sin a$ for an angle may in some cases differ from the tabulated value by 0.01 or 0.02 foot, but such a difference is unimportant in stadia work.

14. Use of Table.—The values given in the stadia reduction table are $R\cos^2 a$ and $R\times\frac{1}{2}\sin 2a$ for a stadia interval factor of 100 and an intercepted distance of 1 foot on the rod. Hence, for a stadia interval factor of 100, the value of $R\cos^2 a$ or $R\times\frac{1}{2}\sin 2a$ for any observed rod interval may be determined by taking from the table the value of 100 $\cos^2 a$ or $100\times\frac{1}{2}\sin 2a$ for the observed vertical angle and multiplying it by the observed rod interval in feet.

EXAMPLE.—Solve the example of Art. 11 by means of the stadia reduction table.

SOLUTION.—The vertical angle is 18°23′ and the number of minutes in this angle is not listed in the table. The value of 100 $\cos^2 a$ for an angle of 18°22′ is given in the table as 90.07, and the value for an angle of 18°24′ is 90.04. Hence, the value for the observed angle is $\frac{90.07+90.04}{2}=90.06$. Since the observed rod interval is 7.21 ft., the required value of $R\cos^2 a$ is

$$7.21\times 90.06=649.3 \text{ ft.}$$

Also, for the given stadia constant of 1 ft., the value of $C \cos a$ is 0.95, which is found near the foot of the column headed Hor. Dist. under the angle 18° and in the horizontal line with $C=1.00$. The required horizontal distance is, therefore,

$$649.3+0.95=650.3 \text{ ft. Ans.}$$

15. In case the stadia interval factor is not equal to 100, the value of $100 \cos^2 a$ or $100\times\frac{1}{2} \sin 2a$ given in the table for the observed vertical angle must be corrected before the multiplication by the rod interval is performed. Whether the factor is greater or less than 100, the first step is to obtain the difference between 100 and the actual factor, and to divide that difference by 100. Then, the tabulated value of $100 \cos^2 a$ or $100\times\frac{1}{2} \sin 2a$ is multiplied by this quotient. If the actual stadia interval factor exceeds 100, the product is added to the tabulated value of $100 \cos^2 a$ or $100\times\frac{1}{2} \sin 2a$; if the actual factor is less than 100, the product is subtracted from that tabulated value. Finally, the resulting sum or difference is multiplied by the observed rod interval and, if desired, the value of $C \cos a$ or $C \sin a$ is added.

EXAMPLE.—The stadia interval factor of a transit is 97.80 and the stadia constant is 1.25 feet. If the vertical angle of the line of sight is 20°40′ and the interval intercepted between the stadia hairs on a vertical rod is 8.24 feet, what are: (*a*) the horizontal distance from the transit to the rod and (*b*) the difference in elevation between the ends of the sight?

SOLUTION.—(*a*) The difference between 100 and the given stadia interval factor is $100-97.80=2.20$ and $2.2\div100=0.022$. Also, the value of $100 \cos^2 a$ in the table for an angle of 20°40′ is 87.54, and $87.54\times0.022 =1.93$. When this value is subtracted from the tabulated value, the result is $87.54-1.93=85.61$. The product of this difference and the observed rod interval is

$$85.61\times8.24=705.4 \text{ ft}$$

From the table, the value of $C \cos a$, for the given stadia constant of 1.25 and the observed vertical angle, is 1.17 ft. Hence, the required horizontal distance is

$$705.4+1.2=707 \text{ ft. Ans.}$$

(*b*) As given in the table, the value of $100\times\frac{1}{2} \sin 2a$ for a vertical angle of 20°40′ is 33.02, and the value of $C \sin a$ is 0.44 ft. Also, $33.02 \times0.022=0.73$, and $33.02-0.73=32.29$. When this value is multiplied by the rod interval, the product is

$$32.29\times8.24=266.07 \text{ ft.}$$

Therefore, the required difference in elevation between the ends of the sight is

$$266.07 + 0.44 = 266.5 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

Solve each of the following examples by use of the stadia reduction table.

1. A sight is taken on a vertical rod with a transit having a stadia interval factor of 100 and a stadia constant of 0.75 foot. If the inclination of the line of sight to the horizontal is 9°21′ and the distance intercepted on the rod between the stadia hairs is 4.28 feet, what are: (*a*) the horizontal distance from the transit to the rod, and (*b*) the difference in elevation between the ends of the sight?

$$\text{Ans.} \begin{cases} (a)\ 417 \text{ ft.} \\ (b)\ 68.7 \text{ ft.} \end{cases}$$

2. In an observation made with a transit for which the stadia interval factor is 101 and the stadia constant is 1 foot, the line of sight makes an angle of 18°14′ with the horizontal and the interval intercepted on a vertical rod between the stadia hairs is 5.47 feet. Determine the horizontal distance from the transit to the rod. Ans. 499 ft.

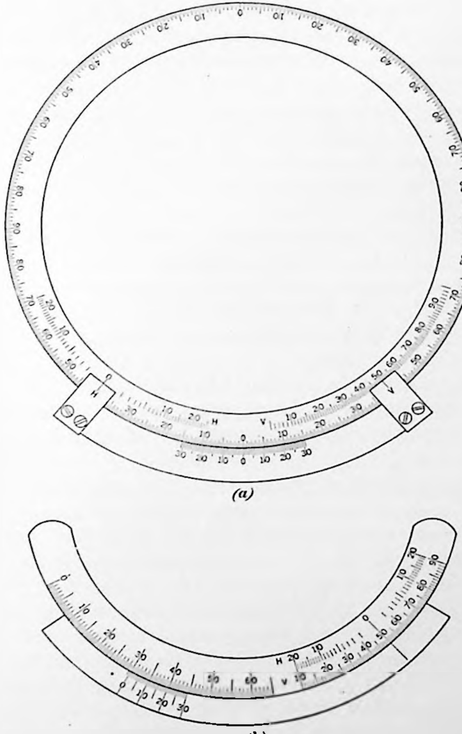
3. For the conditions in the preceding example, what is the difference in elevation between the ends of the sight? Ans. 164.5 ft.

BEAMAN STADIA ARC

16. Theory and Description of Stadia Arc.—If the vertical circle or vertical arc of a transit is equipped with a device known as the Beaman stadia arc, the values of $100 \sin^2 a$ and $100 \times \frac{1}{2} \sin 2a$ can be read directly instead of the angle a that the line of sight makes with the horizontal. The time required to determine horizontal and vertical distances by stadia is thereby greatly reduced.

There are several forms of the Beaman stadia arc, but all are based on the same principles. Two common forms are illustrated in Fig. 4. The type shown in view (*a*) is intended for a transit with a complete vertical circle, and the type shown in view (*b*) is for use with a vertical arc. In either case, vertical angles can be read in the usual manner, if desired, by means of a scale that is graduated in degrees and half-degrees and a vernier which is used with that scale to read minutes.

In view (*a*), the scale for reading vertical angles is along the outer edge of the vertical limb and the vernier is on a small



(a)

(b)

FIG. 4

plate that is fastened to one standard of the transit just below the vertical limb. The scales for determining stadia distances are added to the vertical limb inside the graduations for vertical angles. There are two stadia scales. One of these scales, which is marked *H*, is for corrections to horizontal distances; and the other scale, which is marked *V*, is for differences in

elevation. The index mark for the H scale is also marked H and is on a small metal piece that is screwed to the plate containing the regular vernier at the left-hand end of that plate. Similarly, the index mark for the V scale is marked V and is on a piece at the right-hand end of the plate containing the regular vernier. When the line of sight is horizontal, the 0-graduation on the H scale and the 50-graduation on the V scale should be opposite their respective index marks, as shown in view (*a*).

In view (*b*), the scale for reading vertical angles occupies the left-hand part of the vertical limb and the vernier for that scale is just below the scale on a small plate which is fastened to a standard. The two scales for stadia work are placed one above the other on the right-hand portion of the vertical limb; and there is a common index mark for both scales on the plate containing the regular vernier. As here indicated, this index mark is opposite the 0-graduation on the H scale and the 50-graduation on the V scale when the line of sight is horizontal.

17. Graduations of Stadia Arc.—The reference graduation on the V scale of any stadia arc is marked 50 instead of 0, in order to avoid confusion between angles of elevation and angles of depression. When the reading is more than 50, the vertical angle of the line of sight is an angle of elevation; when the reading is less than 50, the angle is one of depression. The correct value of $100\times\frac{1}{2}\sin 2a$ is found for either condition by taking the difference between 50 and the actual reading.

On both stadia arcs shown in Fig. 4, the graduations on the H scale are numbered from 0 to 20, and the readings indicate the values of $100 \sin^2 a$. However, on some arcs, the central graduation of the H scale is marked 100, and the values decrease in each direction to 80. The reading of such a scale indicates the value of $100 \cos^2 a$.

Where a vertical arc is used in conjunction with a stadia arc, as in Fig. 4 (*b*), the vertical arc is graduated in degrees and parts of a degree, as in the case of the usual vertical arc. The arc has a range from 0° to 60°, but the reading should be 30°, and not 0°, when the line of sight is horizontal. The

reason for this is the same as the reason for marking the center of the V scale 50. For any setting, the true vertical angle is the difference between 30° and the reading of the arc.

18. Computations of Horizontal and Vertical Distances When Stadia Arc Is Used.—When a stadia arc is used, an observation consists in determining the interval intercepted between the stadia hairs on a vertical rod, and also reading the H and V scales of the stadia arc. It is not necessary to observe the vertical angle. For ordinary work with a stadia arc, it is customary to consider that the approximate formulas in Arts. 11 and 12 are accurate enough. The horizontal distance between the two ends of a sight may then be found as follows: If the H scale is graduated as in Fig. 4 and the stadia interval factor is 100, the observed rod interval is multiplied by the H-scale reading, the product is subtracted from 100 times the rod interval, and the stadia constant is added to the result. If the H scale is graduated from 100 to 80 and the stadia interval factor is 100, the H-scale reading is multiplied by the observed rod interval, and the stadia constant is added to the result.

The method of numbering the H-scale graduations from 0 to 20 has the disadvantage that a subtraction must be performed after the scale reading is multiplied by the rod interval; whereas, with a scale numbered from 80 to 100, no subtraction is necessary. On the other hand, the multiplication of the scale reading and the rod interval is usually simpler with readings between 0 and 20 than with readings between 80 and 100.

In case the stadia interval factor is not 100, the first step is to multiply the observed rod interval by the H-scale reading. If the scale graduations are numbered from 80 to 100, the product of the rod interval and the scale reading is multiplied by the ratio of the actual stadia interval factor to 100, and the stadia constant is added. But, if the scale graduations are numbered from 0 to 20, the product of the rod interval and the scale reading is subtracted from 100 times the rod interval, the difference is multiplied by the ratio of the actual stadia interval factor to 100, and the stadia constant is added to the result.

The difference in elevation between the two ends of the sight is found in the following manner: If the stadia interval factor is 100, it is merely necessary to take the product of the observed rod interval and the difference between 50 and the observed reading of the *V* scale. In case the stadia constant is not 100, it is necessary to take the product of the observed rod interval, the difference between 50 and the observed reading of the *V* scale, and the ratio of the actual stadia interval factor to 100.

EXAMPLE 1.—A transit, for which the stadia interval factor is 100 and the stadia constant is 1 foot, is equipped with a Beaman stadia arc. The data obtained in an observation are as follows: The rod interval between the stadia hairs is 5.68 feet, the reading of the *H* scale is 7.3, and the reading of the *V* scale is 24. Determine (*a*) the horizontal distance and (*b*) the difference in elevation between the ends of the sight.

SOLUTION.—(*a*) The product of the rod interval and the *H*-scale reading is $5.68 \times 7.3 = 41$ ft., and 100 times the rod interval is 568 ft. Hence, the required horizontal distance is

$$568 - 41 + 1 = 528 \text{ ft.} \quad \text{Ans.}$$

(*b*) The difference between 50 and the observed *V*-scale reading is $50 - 24 = 26$, and the required difference in elevation is

$$5.68 \times 26 = 147.7 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 2.—Solve the preceding example, assuming that the stadia interval factor is 101.

SOLUTION.—(*a*) In this case, the first step is to subtract the product of 5.68 and 7.3, or 41, from 100 times the rod interval, or 568. The difference is $568 - 41 = 527$ ft. The required horizontal distance is then found by multiplying this difference by $\frac{101}{100}$ and adding the stadia constant. Thus,

$$527 \times \frac{101}{100} + 1 = 533 \text{ ft.} \quad \text{Ans.}$$

(*b*) The required difference in elevation is determined simply by multiplying the difference in elevation found in the preceding example by $\frac{101}{100}$. Thus,

$$147.7 \times \frac{101}{100} = 149.2 \text{ ft.} \quad \text{Ans.}$$

19. For very accurate work and especially for large vertical angles, it may be necessary to make allowance for the stadia constant in determining the difference in elevation between the

ends of the sight. The calculations may then be made in the following manner: The first step is to divide the stadia constant by the stadia interval factor and to add the quotient to the observed rod interval. The next step is to multiply this sum by the difference between 50 and the observed reading of the V scale. If the stadia interval factor is 100, this product is the required result. Otherwise, this product is multiplied by the ratio of the actual stadia interval factor to 100. The vertical distance thus obtained is the same as that determined by formula 2, Art. 12.

EXAMPLE.—Determine the difference in elevation between the ends of the sight for which the data are as given in example 2 of Art. 18, allowance being made for the stadia constant.

SOLUTION.—The quotient of the stadia constant and the stadia interval factor is $1 \div 101 = 0.01$ ft. and the sum of this value and the observed rod interval is $5.68 + 0.01 = 5.69$ ft. Since the difference between 50 and the observed V-scale reading is $50 - 24 = 26$, the required difference in elevation is

$$5.69 \times 26 \times \frac{101}{100} = 149.4 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. For a certain sight, the interval intercepted between the stadia hairs on a vertical rod was 4.72 feet and the reading of the V scale of a Beaman stadia arc was 66. If the stadia interval factor was 100 and the stadia constant was 1 foot, what was the difference in elevation between the ends of the sight? Consider the effect of the stadia constant.

Ans. 75.7 ft.

2. What was the horizontal distance between the center of the instrument and the rod in the preceding example, if the H-scale reading of the stadia arc was 2.6? Ans. 461 ft.

3. Assuming that the stadia interval factor in the preceding examples was 98.5, determine: (*a*) the difference in elevation and (*b*) the horizontal distance between the ends of the sight.

Ans. $\begin{cases} (a)\ 74.6 \text{ ft.} \\ (b)\ 454 \text{ ft.} \end{cases}$

FIELD OPERATIONS IN STADIA WORK

PRACTICAL CONSIDERATIONS IN STADIA WORK

20. Field Party.—The party making stadia measurements may consist of as few as two men or as many as eight men or even more. Although progress may be slow, the work can be done by a transitman and one rodman. The transitman then serves also as recorder, and the rodman not only holds the rod but performs the duties of an axman as well. Where great speed is desired or the character of the country makes the employment of a larger party necessary, a recorder and the required number of rodmen and axmen should be added to the party.

21. Stadia Rods.—An ordinary self-reading leveling rod is often used for stadia work where the sights are comparatively short. However, more satisfactory and more rapid work can be done with a rod that is made especially for the purpose. A stadia rod is generally between 10 and 15 feet long and between 3 and 5 inches wide. Such a rod is often made in two equal sections, which are hinged together so that the rod can be folded for convenience in transportation. The two sections are sometimes joined by clamps when in use. Occasionally, a stadia rod is made by temporarily fastening to the face of a plain board a strip of flexible material on which graduations have been printed or painted. The strip may be detached from the board and rolled up when the rod is not in use.

Rods that are graduated according to certain patterns are obtainable from manufacturers of surveying instruments, but many stadia rods are graduated according to the specifications of the individual surveyor. Numerous patterns have therefore been designed, the principal aim being to use markings of such shape that readings can be easily and quickly made by the instrumentman.

22. Four typical patterns of graduation marks on stadia rods are illustrated in Fig. 5. In each of these cases, the rod is divided into feet by certain distinctive graduation marks and is further

subdivided into decimals of a foot by other graduation marks; but the meter and the yard are also used as the units of measurement on stadia rods. There are no numbers on these rods, and the number of feet in the distance intercepted between the stadia hairs must be determined by counting the foot intervals.

The graduation marks representing tenths of a foot in view (*a*) are located at the intersections of the sloping sides of the respective blackened triangles. The total height of any triangle is 0.1 foot, and each inclined side of the triangle covers a

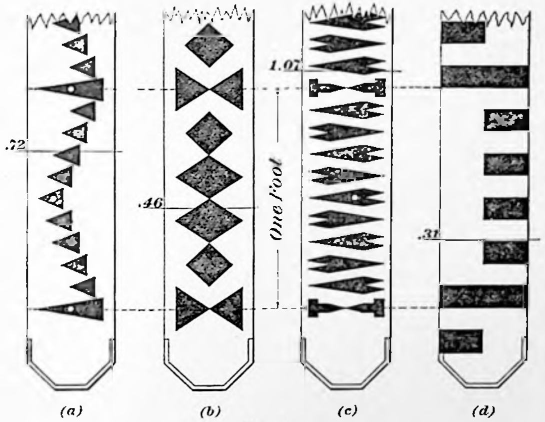


FIG. 5

height of 0.05 foot. The top or bottom of a triangle is, therefore, at the middle of a tenth of a foot, and the reading of the rod to hundredths of a foot is thus facilitated. A reading of 0.72 foot is indicated by the light full horizontal line representing the image of a stadia hair.

On the type of rod shown in view (*b*), each corner of a diamond-shaped figure is at a graduation representing some tenth of a foot. Thus, each side of such a figure covers a height of 0.1 foot and the number of hundredths of a foot in a particular rod reading must be estimated by subdividing a tenth

by eye. The reading indicated in this view by the light line representing the image of a stadia hair is 0.46 foot.

In view (*c*), the graduations representing tenths of a foot are located at the intersections of the long inclined lines. Any other intersection of two slopes is a graduation mark representing a multiple of 0.02 foot. A reading of 1.07 feet is indicated by the position of the image of the stadia hair.

The graduations in view (*d*) are somewhat like those on a leveling rod of the Philadelphia type, but each blackened space has a height of 0.1 foot on the stadia rod and of only 0.01 foot on the leveling rod. On this type of stadia rod, the number of hundredths of a foot in a reading is estimated by subdividing a tenth of a foot by eye. The stadia hair in view (*d*) indicates a reading of 0.31 foot.

A good method of graduating a stadia rod that is particularly suitable for long sights is shown in Fig. 6. The rod is graduated in feet and tenths of a foot, and the foot intervals are shown by the numbers. The total height of each number is 0.5 foot. Also, the thickness of each portion of a number and of each black or white division between numbers is 0.1 foot. Any horizontal line in a number can be used as the foot mark, provided corresponding lines are used in all readings. This pattern has the special advantage that the number of feet in the distance intercepted between the stadia hairs can be readily determined without actually counting the foot intervals.

Fig. 6

23. Where numerous points are to be located by stadia with an instrument whose stadia interval factor is not exactly 100, it may be desirable to use a specially graduated stadia rod on which each main division is equal to 100 feet divided by the particular stadia interval factor. Readings made on such a rod are recorded as if each main division were 1 foot in length, and the horizontal and vertical distances are then determined as if the value of the stadia interval

factor were 100. A rod that has divisions of special length can be employed only with the instrument for which it was graduated. If such a rod were used for direct leveling, the elevations determined with it would obviously be incorrect. Therefore, the practice of graduating a rod to correspond to a certain stadia interval factor is no longer common.

MAKING OBSERVATIONS AND COMPUTING RESULTS

24. Location of Points.—A point over which the instrument is set for the purpose of taking stadia readings on other points is called an *instrument point*. A point on which the rod is held for observation is called a *stadia point*. Each stadia point is generally located, with respect to the instrument point from which it is sighted, by the direction and length of the line from the instrument point to the stadia point. In the explanations immediately following, it will be assumed that a transit is used. Observations with a plane table will be described later.

In a transit-stadia survey, the direction of the line of sight is usually determined by its azimuth, which is read from the horizontal limb of the instrument. The distance to the stadia point is obtained from the interval intercepted between the stadia hairs on the rod and either the vertical angle, which is read from the vertical limb of the transit, or the stadia-arc reading. If it is desired to determine also the elevation of the stadia point, the difference in elevation between the ends of the sight may be determined from the data of the observation; and this difference may be used with other data to compute the required elevation.

25. Determination of Rod Interval.—When it is desired to determine a distance by stadia, the first step is to find the rod interval, or the difference between the readings of the upper and lower stadia hairs on a vertical rod. The rod should be vertical, or plumb, when the rod readings are observed, because any inclination of the rod affects the interval intercepted on it between the stadia hairs. The rod can be plumbed accurately by holding a rod level against the side of the rod and changing the inclination of the rod so as to center the bubble or bubbles of the level.

The usual procedure in determining the rod interval is as follows: The rod is held vertically on the stadia point and the telescope is directed toward the rod with the line of sight inclined approximately parallel to the straight line from the instrument point to the stadia point. The telescope is then raised or lowered, preferably by means of the tangent screw that controls the vertical movement of the telescope, so that the lower stadia hair is exactly on some foot-mark on the rod. The observation for the rod interval is completed by noting the rod reading at the upper stadia hair, taking the difference between the readings of the upper and lower hairs, and recording that difference. Thus, if the lower hair is made to coincide with the 4-foot mark on the rod and the upper hair intersects the rod at 6.35 feet, the rod interval is $6.35-4.00=2.35$ feet. Often it is assumed that the stadia interval factor is 100, and the value of R, or in this case 235 feet, is recorded instead of the actual rod interval or 2.35.

If the rod interval is observed with the telescope inclined so that neither stadia hair cuts the rod at a foot-mark, an extra amount of time and care is required in order to determine the rod readings of the stadia hairs and the difference between those readings.

26. Methods of Determining Inclination of Line of Sight. After the rod interval has been observed for a particular stadia sight, the next step is to determine the inclination of the line of sight. When a stadia arc is not used, there are two general methods for determining the vertical angle. In one method, the telescope is rotated vertically from its position for determining the rod interval until the middle hair comes on a foot-mark. In the other method, the telescope is rotated so that the middle hair cuts the rod at a point whose height above the stadia point is equal to the height of the center of the telescope above the instrument point. This height, which is called the *height of the instrument*, may be measured by means of a pocket tape or a graduated rod. It is important to bear in mind that the term height of instrument does not have the same meaning in stadia work as it does in direct leveling, where

it is the height of the center of the instrument above some specified reference plane.

When the vertical angle is determined by setting the middle hair at the height of the instrument, the line of sight is made parallel to the straight line joining the instrument point and the stadia point. For example, if the distance *OD* in Fig. 3 is made equal to the distance *ME*, the line of sight *ED* is parallel to the line *MO*. The difference in elevation between the instrument point and the stadia point is then equal to the difference in elevation between the ends of the sight, or the vertical distance *DG* in Fig. 3 is equal to the distance *ON*. The calculations for determining the elevation of the stadia point are thus somewhat simpler than those required when the line of sight is not parallel to the line joining the instrument and stadia points. However, in order to set the middle hair at the height of the instrument, it is necessary to measure that height; whereas this measurement need not be made if the middle hair is set to a foot-mark. Also, a portion of the rod is sometimes hidden or obscured by leaves or other obstructions so that it is not convenient to set the middle hair at the height of the instrument, and the vertical angle for some other rod reading must then be observed. Any rod reading would be satisfactory in such a case, but it is preferable to set the middle hair either at some convenient foot-mark or at a whole number of feet above or below the height of the instrument.

27. In case the stadia arc is used, the computation of the difference in elevation between the ends of the sight is simplified if the telescope is inclined so that some graduation mark on the *V* scale of the arc coincides with the index mark.

No matter which method of determining the inclination of the line of sight is employed, it is usually necessary to set the telescope at one inclination for determining the rod interval and at a slightly different inclination for determining the vertical angle or the stadia-arc reading. If the rod is not plumb when the middle hair is set or when its reading is observed, the computed elevation will be in error, but usually this error is negligible.

28. Details of Observations Without Stadia Arc.—When the stadia work is being done without a stadia arc, the rod need be held on the stadia point only long enough to observe the rod interval and to set the middle hair on the proper rod graduation for determining the vertical angle. The azimuth and the vertical angle are read and recorded while the rodman is moving to the next stadia point. An angle of elevation is indicated by writing the sign + before its value in degrees and minutes, and an angle of depression is indicated by the sign −. Except when the middle hair is set at the height of the instrument, the reading of the middle hair is also entered in the notes. The form for recording the field notes and the procedure in calculating the elevations of the various points depend on the general method that is used for determining the inclination of the line of sight.

29. Details of Observations With Stadia Arc.—When a stadia arc is used, the next operation after determining the rod interval is to observe roughly the reading of the *V* scale of the stadia arc and to alter the inclination of the telescope, by means of the tangent screw that controls vertical motion, so that the nearest graduation mark on the *V* scale coincides exactly with the index mark. For this position of the telescope, the reading of the *V* scale is recorded; the reading on the rod marked by the middle hair is observed and entered in the notes; and the azimuth and the reading of the *H* scale of the stadia arc are observed and recorded.

30. Determination of Horizontal Distances.—The horizontal distance from the instrument point to any stadia point can be readily determined from the observed rod interval and either the vertical angle or the reading of the *H* scale of a stadia arc. If the vertical angle is observed, it is merely necessary to apply one of the formulas of Art. 11 or to use the stadia reduction table. If a stadia arc is used, the method described in Art. 18 is employed.

31. Elevations When Middle Hair Is Set at Foot-Mark. If the vertical angle is determined by setting the middle hair

at a convenient foot-mark, the elevations of the various points are usually calculated in the following manner: The difference between the elevation of the line of sight at the instrument point and the known elevation of a stadia point is determined from the observed data for a backsight on that stadia point. Also, the difference between the known elevation of the line of sight at the instrument point and the elevation of any stadia point is found from the observed data for a foresight on the stadia point. In either case, the difference between the elevation of the line of sight at the instrument point and the elevation of the point on which the rod is held is the algebraic sum of the following two distances: (1) the rod reading of the middle hair, and (2) the difference in elevation between the ends of the sight, or between the center of the telescope and the point on the rod at which it is cut by the middle hair.

When the rod is held on a point of known elevation and it is desired to determine the elevation of the line of sight at the instrument point, the rod reading of the middle hair is considered positive, and the sign of the difference in elevation between the ends of the sight is opposite to the sign of the observed vertical angle. The algebraic sum of these two distances is then added algebraically to the elevation of the point on which the rod is held, in order to obtain the required elevation of the line of sight at the instrument point.

In case the rod is held on a point whose elevation is desired, the reading of the middle hair is considered negative, the difference in elevation between the ends of the sight is given the same sign as the observed vertical angle, and the algebraic sum of these two distances is added algebraically to the elevation of the line of sight at the instrument point.

Example.—Stadia work is being done with a transit for which the stadia interval factor is 100 and the stadia constant is 1 foot. When a backsight is taken on a point whose elevation is 210.8 feet, the rod interval is 6.35 feet; and, with the middle hair set at 5.0 feet on the rod, the vertical angle is $-3°30'$. For a foresight to another point, the rod interval is 3.84 feet; and, with the middle hair reading 5.0 feet on the rod, the vertical angle is $+2°38'$. Determine (*a*) the elevation of the line of sight at the instrument point and (*b*) the elevation of the point on which the foresight is taken.

Solution.—(*a*) From the stadia reduction table, the value of $100 \times \frac{1}{2} \sin 2a$ for a vertical angle of 3°30′ is 6.09 and the value of $C \sin a$ for a stadia constant of 1 ft. is 0.06 ft. Hence, the difference in elevation between the ends of the sight for the first point is

$$6.09 \times 6.35 + 0.06 = 38.7 \text{ ft.}$$

Since it is required to find the elevation of the line of sight, the reading of the middle hair is taken as +5.0. Also, for a negative vertical angle, the difference in elevation between the ends of the sight is positive; in this case, the difference in elevation is +38.7. The algebraic sum of the two distances is

$$+5.0 + 38.7 = +43.7 \text{ ft.}$$

and the required elevation of the line of sight at the instrument point is

$$210.8 + 43.7 = 254.5 \text{ ft. Ans.}$$

(*b*) In this case, the difference in elevation between the ends of the sight is

$$4.59 \times 3.84 + 0.04 = 17.7 \text{ ft.}$$

This distance is considered positive because the vertical angle is positive, and the reading of the middle hair is taken as −5.0 ft. The algebraic sum of the two distances is

$$+17.7 - 5.0 = +12.7 \text{ ft.}$$

and the required elevation of the stadia point is

$$254.5 + 12.7 = 267.2 \text{ ft. Ans.}$$

32. Elevations When Middle Hair Is Set at Height of Instrument.—If the vertical angle is read when the middle hair cuts the rod at the height of instrument, the line of sight is parallel to the line joining the instrument point and the stadia point. The difference in elevation between the ends of the sight, which may be found either by one of the formulas of Art. 12 or by use of the stadia reduction table, is then also the difference in elevation between the instrument point and the stadia point.

When the sight is taken on a point of known elevation in order to determine the elevation of the instrument point, the computed difference in elevation is added to the given elevation if the vertical angle is negative or is subtracted if the angle is positive. When the sight is taken from an instrument point of known elevation to a stadia point whose elevation is desired, the computed difference in elevation is added to the given eleva-

tion if the vertical angle is positive and is subtracted if the angle is negative.

EXAMPLE.—The transit used in a stadia survey has a stadia interval factor of 100 and a stadia constant of 1 foot. The rod interval between the stadia hairs for a certain stadia point is 3.69 feet and, when the middle hair is set so that it intersects the rod at the height of instrument, the vertical angle is found to be $+7°20'$. If the elevation of the instrument point is 217.4 feet, what is the elevation of the stadia point?

SOLUTION.—The difference in elevation between the instrument point and the stadia point is

$$12.66 \times 3.69 + 0.13 = 46.8 \text{ ft.}$$

Since the observed vertical angle is positive, or the angle is an angle of elevation, the required elevation is found by adding 46.8 ft. to the elevation of the instrument point. Thus, it is

$$217.4 + 46.8 = 264.2 \text{ ft.} \quad \text{Ans.}$$

33. Elevations When Middle Hair Cannot Be Set at Height of Instrument.—If the general method is to measure the vertical angles with the middle hair set at the height of instrument and only an occasional sight is taken with a different reading of the middle hair, it is generally desirable to determine for every sight the difference in elevation between the instrument point and the point on which the rod is held. To find this difference when the reading of the middle hair is not equal to the height of instrument, the difference in elevation between the ends of the sight is added to the height of instrument if the vertical angle is positive or is subtracted from that height if the vertical angle is negative; and the actual rod reading of the middle hair is subtracted from the result thus obtained.

Whether the difference in elevation between the instrument point and the stadia point should be added to or subtracted from the given elevation depends on the conditions. If the sight is taken on a point of known elevation and it is required to determine the elevation of the instrument point, the computed difference in elevation is subtracted algebraically from the known elevation. If the elevation of the stadia point is required, the computed difference in elevation is added algebraically to the known elevation of the instrument point.

EXAMPLE.—The stadia interval factor is 100; the stadia constant is 1 foot; the elevation of the instrument point is 248.1 feet; the height of instrument is 4.8 feet; the rod interval is 6.35 feet; and, with the middle hair reading 6.8 feet on the rod, the observed vertical angle is $-3°30'$. What is the elevation of the stadia point?

SOLUTION.—The difference in elevation between the ends of the sight is

$$6.09 \times 6.35 + 0.06 = 38.7 \text{ ft.}$$

In this case, the vertical angle is negative and this distance is subtracted from the height of instrument. The difference is

$$4.8 - 38.7 = -33.9 \text{ ft.}$$

The difference in elevation between the instrument point and the stadia point is

$$-33.9 - 6.8 = -40.7 \text{ ft.}$$

and the required elevation of the stadia point is

$$248.1 - 40.7 = 207.4 \text{ ft.} \quad \text{Ans.}$$

34. Elevations When Stadia Arc is Used.—If a stadia arc is used, the difference in elevation between the ends of the sight can be found from the rod interval and the reading of the *V* scale in the manner described in Art. 18. The elevation of the line of sight at the instrument point or the elevation of the stadia point can then be computed from this difference in elevation and other data in the manner described in Art. 31. In determining the sign of the difference in elevation between the ends of the sight when the stadia arc is used, it should be kept in mind that the vertical angle is positive when the *V*-scale reading is more than 50 and is negative when that reading is less than 50.

EXAMPLE.—The stadia interval factor is 100; the stadia constant is 1 foot; the elevation of the line of sight at the instrument point is 566.7 feet; the rod interval is 5.75 feet; and, when the *V* scale of the stadia arc is set to read 40, the reading of the middle hair on the rod is 4.8 feet. What is the elevation of the stadia point?

SOLUTION.—If the stadia constant is neglected, the difference in elevation between the ends of the sight is

$$5.75 \times (50 - 40) = 57.5 \text{ ft.}$$

Since the *V*-scale reading is less than 50, the vertical angle is negative and this difference in elevation is taken as -57.5 ft. The algebraic sum of this difference and the reading of the middle hair is

$$-4.8 - 57.5 = -62.3 \text{ ft.}$$

and the elevation of the stadia point is

$$566.7-62.3=504.4 \text{ ft.} \quad \text{Ans.}$$

35. Observation for Very Long Sight.—Sometimes the length of the sight exceeds 100 times the total height of the rod that is used, and the interval between the stadia hairs at the stadia point cannot be observed directly. The desired interval may then be determined in the following manner: The telescope is inclined first so that the rod interval between the lower stadia hair and the middle hair can be observed, and then so that the rod interval between the middle hair and the upper stadia hair can be observed. The sum of these two intervals is taken as the total rod interval. If it has been found by test that the middle hair is exactly midway between the two stadia hairs, it may be sufficient to observe only the interval between the middle hair and either stadia hair, and to take twice that distance as the required rod interval.

36. Observation Where Line of Sight May Be Horizontal. Where the difference in elevation between the instrument point and the stadia point is only a few feet, the middle hair will cut the rod when the line of sight is horizontal. In such a case, it is preferable to level the telescope as for direct leveling and to observe and record the rod reading at the middle hair, rather than to set the middle hair at the height of instrument or some foot-mark and to observe the corresponding vertical angle, because the determination of the horizontal and vertical distances is greatly simplified. The horizontal distance from the instrument to the rod is then equal to $R+C$. When the general method is to measure the vertical angle with the middle hair at a foot-mark, the elevation of the line of sight at the instrument point can be found from the known elevation of a stadia point by adding the rod reading on the stadia point to that known elevation; or the elevation of a stadia point can be found from the known elevation of the line of sight at the instrument point by subtracting the rod reading from the known elevation.

When the general method is to measure vertical angles with the middle hair at the height of the instrument, the difference

in elevation between the instrument point and the point on which the rod is held may be found simply by subtracting the actual rod reading of the middle hair from the height of the instrument. If it is required to find the elevation of the instrument point from the elevation of the other point, the difference in elevation is subtracted algebraically from the given elevation. If the elevation of the stadia point is desired, the difference in elevation is added algebraically to the elevation of the instrument point.

EXAMPLE.—The stadia interval factor is 100.5; the stadia constant is 1 foot; the elevation of the instrument point is 1,876.3 feet; the height of instrument is 5.0 feet; the rod interval is 4.10 feet; and, when the line of sight is horizontal, the middle hair cuts the rod at a height of 2.1 feet. Determine (*a*) the horizontal distance from the instrument point to the stadia point, and (*b*) the elevation of the stadia point.

SOLUTION.—(*a*) The required horizontal distance is

$$R+C=4.10\times 100.5+1=413 \text{ ft.} \quad \text{Ans.}$$

(*b*) The difference between the height of the instrument and the rod reading of the middle hair is $5.0-2.1=+2.9$ ft. Since this difference is positive, the elevation of the stadia point is

$$1{,}876.3+2.9=1{,}879.2 \text{ ft.} \quad \text{Ans.}$$

37. Observations Where Elevations Are Not Required.—In some surveys it is not necessary to determine differences of elevation by stadia. Since the value of $\sin^2 a$ is very small for sights that are nearly horizontal, and since the horizontal distance is affected but slightly by a difference of a few minutes in the vertical angle, it is customary in such a survey to disregard vertical angles that are less than about 3° and to record larger vertical angles to the nearest 10 minutes without reading the vernier accurately. Also, no distinction is made between angles of elevation and angles of depression.

38. Stepping Method.—Where the angle of inclination of the line of sight from the instrument point to a stadia point is comparatively small, but it is not possible to read the rod with the telescope horizontal, the elevation of the stadia point can sometimes be found conveniently by a method that is known as the stepping method. This method has the advantage that

no vertical angle need be read and the calculations are very simple.

The principle on which the stepping method is based is illustrated in Fig. 7. In order to explain the method more clearly, the differences in elevation shown in the illustration are greatly exaggerated; actually, these differences are comparatively small. First, the interval intercepted between the stadia hairs on a rod held vertically on the stadia point is

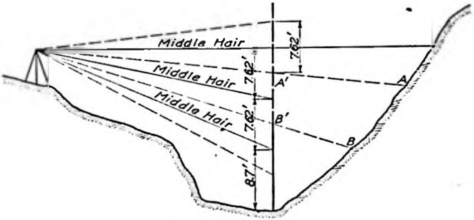


FIG. 7

determined in the usual manner. In the illustration, this interval is shown to be 7.62 feet. Then, the line of sight is made horizontal by leveling the telescope, and a point, such as A, at which the landscape is cut by the lower stadia hair is noted mentally. Such a point may be a distinguishable natural mark on a rock, a flower, or the tip of a leaf. If the observer keeps the point constantly in view, he can usually relocate it with sufficient accuracy for the next setting of the telescope.

As soon as the observer has fixed in his mind a reference point at A to mark the position of the lower hair, he lowers the line of sight, by means of the tangent screw that controls the vertical motion of the telescope, until the upper stadia hair passes through the same reference point at A. The second position of the lower hair, as B, is then noted on the landscape. The next operation is to lower the telescope again so as to bring the upper hair through the point B. The stepping is continued until the middle hair of the transit comes on the rod held on the stadia point. For the conditions represented in

Fig. 7, the rod reading of the middle hair is 8.7 feet when the upper stadia hair is at *B*. In this case, the difference in elevation between the ends of the sight after the second step is evidently equal to $2 \times 7.62 = 15.2$ feet. The elevation of the stadia point can be found from this difference in elevation, the elevation of the line of sight at the instrument point, and the rod reading of the middle hair in the manner explained in Art. 31.

The foregoing explanation is for a case where the stadia point is lower than the instrument. A similar method may be adopted when the stadia point is higher than the instrument. Thus, in Fig. 8, the rod interval between the stadia

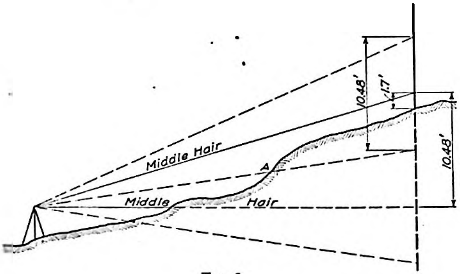


FIG. 8

hairs is determined and the line of sight is made horizontal, as in the previous case. But, for each position of the telescope, the reference point on the landscape is located by the upper stadia hair and the lower hair is brought to the reference point for the previous sight. For the conditions illustrated in Fig. 8, the middle hair cuts the rod when the lower hair passes through the first reference point *A* established by the position of the upper hair, and hence only one step is necessary. The difference in elevation between the ends of the sight when the middle hair is in its final position is therefore equal to the rod interval intercepted between the stadia hairs, or 10.48 feet.

39. When the stepping method is used for determining the elevation of a stadia point, the difference in elevation between

the ends of the sight with the middle hair in its final position is always equal to the product of the number of steps and the interval intercepted between the stadia hairs on the rod held at the stadia point. After this difference in elevation has been determined, the elevation of the stadia point can be computed in the manner described in Art. 31.

In the stepping method, it is assumed that the rod interval intercepted between the stadia hairs is the same for all vertical angles. Thus, in Fig. 7, the rod interval at the stadia point is assumed to be the same whether the lower stadia hair is at A' or the upper hair is at B'. Since such an assumption is not sufficiently accurate to give the exact difference in elevation where the vertical angle is much more than 1°, the stepping method should not be used in fairly accurate work where more than three steps are required. More steps would be permissible for rough work; but, where too many steps would be needed, the field work would take too much time and the stepping method would not be economical.

Example.—The elevation of the line of sight at the instrument point is 681.3 feet and the interval intercepted between the stadia hairs on a rod held at a stadia point is 5.84 feet. In order to determine the elevation of the stadia point by the stepping method, the line of sight has to be elevated. If, after two steps, the middle hair cuts the rod on the stadia point at a height of 4.1 feet, what is the elevation of the stadia point?

Solution.—The difference in elevation between the ends of the sight after the second step is $2\times5.84=+11.7$ ft. Since $+11.7-4.1=+7.6$ ft., the required elevation is

$$681.3+7.6=688.9 \text{ ft. Ans.}$$

EXAMPLES FOR PRACTICE

1. The elevation of an instrument point was 308.9 feet and the height of instrument was 5.0 feet. Also, the interval between the stadia hairs on a rod held at a certain stadia point was found to be 6.22 feet and, when the middle hair read 5.0 feet on the rod, the vertical angle was $-12°14'$. If the stadia interval factor was 100 and the stadia constant was 1.25 feet, what was the elevation of the stadia point?

Ans. 179.8 ft.

2. From the instrument point in the preceding example, a sight was taken to a second stadia point at which the rod interval between the stadia hairs was 8.13 feet. When the middle hair read 3.0 feet on the

rod, the vertical angle was observed as $+5°10'$. Determine the elevation of the stadia point. Ans. 383.9 ft.

3. An instrument having a stadia interval factor of 100 and an instrument constant of 1 foot was set up at a point of unknown elevation. When a backsight was taken on a point having an elevation of 275.0 feet, the rod interval was 3.91 feet; and, when the middle hair was set at 6.0 feet on the rod, the vertical angle was $+4°46'$. What was the elevation of the line of sight at the instrument point? Ans. 248.5 ft.

4. In order to find the elevation of a stadia point from the set-up in example 3, the stepping method was used. The rod interval between the stadia hairs was 6.73 feet and, after the line of sight was depressed from the horizontal for one step, the reading of the middle hair on the rod was 9.4 feet. What was the elevation of the stadia point?
Ans. 232.4 ft.

5. A transit that is equipped with a stadia arc and has a stadia interval factor of 100 is set up so that the elevation of the line of sight at the instrument point is 581.7 feet. The observed rod interval on a stadia point is 2.85 feet and, when the stadia arc is set to read 64, the reading of the middle hair on the rod is 4.3 feet. What is the elevation of the stadia point? Ans. 617.3 ft.

ACCURACY OF STADIA WORK

40. Errors in Stadia Measurements.—The accuracy of stadia measurements depends largely on the instrument and the rod that are used and on the condition of the atmosphere. For sights up to 400 feet in length, it is usually possible to obtain the rod interval between the stadia hairs to within 0.01 foot of the correct reading. Consequently, if the correct stadia interval factor is used and the graduations on the rod are accurate, the error in the computed horizontal distance from the instrument to the rod should not exceed 1 foot. An accurate value of the stadia interval factor may be determined by test, as explained in Art. 9. For work of ordinary precision, errors due to imperfections in the graduation of the rod will be negligible in the case of a rod that is well constructed and accurately graduated.

Even if the stadia interval factor and the rod graduations are correct, accurate results cannot be obtained unless the observation is made when the condition of the atmosphere is satisfactory and the rod is held plumb. In the middle of the

day, rays of light through the two stadia hairs are likely to be deflected unequally by refraction, because the ray through the lower hair passes closer to the ground than does the ray through the upper hair. Also, on hot days boiling air or heat waves are apt to cause inaccurate rod readings, and in windy weather vibration of the instrument under the force of the wind is a possible source of error. When the rod is not held plumb and the vertical angle is large, the rod interval that is observed on the inclined rod may be greatly in error. In the case of long sights, the accuracy of the rod interval is dependent also on the magnifying power of the telescope, the coarseness of the stadia hairs, and the pattern of the rod. Under reasonably favorable conditions, the error in horizontal distance, as determined by stadia, should not exceed 10 feet in a length of 1,000 feet.

The principal causes of errors in elevations of stadia points are errors in vertical angles and errors in determining the rod interval. The effect of an error in vertical angle is practically independent of the size of the angle but is proportional to the length of the sight, which is in turn proportional to the rod interval. The error in elevation caused by the rod being out of plumb increases rapidly as the vertical angle and the rod interval increase.

Errors introduced in observing the rod interval because the rod cannot be read exactly will tend to balance one another. In other words, the interval will probably be too great in some cases and too small in other cases. No general rule can be given for the maximum length of sight that is permissible, because this limit depends on many conditions. However, the following facts should be kept in mind. In all cases for which the rod can be read correctly to the nearest hundredth of a foot, the probable error in the rod interval is independent of the length of sight. The length of the longest sight that will permit reading the rod with the desired accuracy depends on the topographic conditions, the characteristics of the instrument and rod that are used, and the weather. Longer sights are possible when the rod is held so as to face the sun than when it is turned away from the sun or is in the shade.

41. Methods of Minimizing Unavoidable Errors.—Besides being careful to avoid mistakes, the instrumentman on a stadia survey should take certain precautions to keep the unavoidable errors as small as possible. Thus, when the stadia hairs appear to cover an appreciable space on the rod, the instrumentman should read the positions of the tops of the hairs rather than attempt to estimate the readings at their centers. On important sights, the readings of all three horizontal hairs should be observed. If the stadia hairs are equidistant from the middle hair, the reading of the middle hair should be the average of the readings of the stadia hairs.

Where the lengths of the courses of a traverse are being determined by stadia, observations should be made from both ends of each line. The mean of the two values thus obtained for a course should be taken as the true length. If elevations are also being found, the difference in elevation between the two ends of each course should likewise be determined from both ends, and the average of the computed results should be used.

TRANSIT-STADIA SURVEYS FOR LOCATING TOPOGRAPHY

42. Outline of Method.—The most common use of stadia is in a survey for locating features of topography, in order to obtain data for a topographic map. In such a survey, the details of topography are located from the corners of a traverse. This traverse may be run in various ways, but a traverse run by transit and stadia is well adapted for the purpose. The direction of each course of the traverse is usually established by its azimuth, and both the lengths of the courses and the elevations of the corners are determined by stadia. While the transit is set up at any corner of the traverse, each object of topography in the vicinity of that corner is located by observing the azimuth, stadia reading, and other values needed for determining the horizontal distance and difference in elevation between the traverse corner and the object.

43. Typical Notes for Instrument Without Stadia Arc.—In Fig. 9 is shown a good form of notes for a topographic survey

which is made with a transit that is not equipped with a stadia arc and in which vertical angles are usually measured when the reading of the middle hair is equal to the height of instrument. In the first column, which is headed *Ver. A*, is entered the azimuth, or the reading of vernier *A*. The second column is for the observed stadia distance, which is taken as 100 times the interval intercepted on the rod between the stadia hairs. The value in the third column is the vertical angle. In the fourth column, headed *Rod Read.*, is placed the reading of the middle hair of the transit in case that reading is not the same

Ver. A	Stadia Dist.	Vert. ∠	Rod Read.	Diff. Elev.	Elev.	Cor. Hor. Dist.	Object
	At	Sta. 1.	H.	I.=5.1.	Elev.	=	
24°12′	437	−3°18′					Sta. 2
67 15	332	−2 11			877.3		B.M.#27
173 35	625	−4 33					Foot of slope
182 45	658	+1 19					Top of hill
189 10	432	+2 46					" " "
193 15	420	−3 44	7.1				Foot of slope
24 12							Sta. 2
	At	Sta. 2.	H.	I.=4.9.	Elev.	=	
204 12	436	+3 17					Sta. 1
47 43	578	−2 14					Sta. 3
155 55	446	+4 36	8.9				Top of hill
152 25	427	+0 18	5.9				Foot of slope
127 45	218	— —	9.3				" " "
204 12							Sta. 1

FIG. 9

as the height of the instrument above the ground. The fifth, sixth, and seventh columns are headed *Diff. Elev.*, *Elev.*, and *Cor. Hor. Dist.*, respectively. In them are entered the computed difference in elevation between the point of set-up and the point sighted on, the computed elevation of the point sighted on, and the computed corrected horizontal distance from the point of set-up to the point sighted on. In the last column, which is headed *Object*, is given the description of the point sighted on. For each set-up, the identification of the transit point, the height of instrument (H. I.), and the eleva-

tion of the transit point are entered in the notes across the page in the manner indicated. Only the given values and the values observed in the field are shown in Fig. 9. The computed values have not been entered in these notes.

44. Details of Field Work.—In making a topographic survey, the transit is set up at the first corner of the traverse, and the height of instrument is measured. The transit is then oriented in azimuth by bringing the line of sight into some convenient reference line with the vernier set at the known or assumed azimuth of that reference line. Also, the elevation of the transit point is established. In the survey for which the notes are given in Fig. 9, the first transit point was designated as Sta. 1, and the height of instrument, or H. I., at that point was 5.1 feet. The elevation of Sta. 1 was not known at the beginning of the survey. The transit was oriented at Sta. 1 by sighting to Sta. 2 with the vernier reading the azimuth of the course between those stations, which was known to be 24°12′ from a previous survey. While the line of sight was directed to Sta. 2, the stadia distance and the vertical angle were also observed, and were entered in the notes. It is important that the sign of the vertical angle should be included in the notes, in order to show whether the angle is an angle of elevation or an angle of depression.

The second sight from Sta. 1 was to a bench mark, designated as B.M. # 27. The known elevation of this bench mark was inserted in the column of the notes headed *Elev.*; and the stadia distance, azimuth, and vertical angle for the sight from Sta. 1 to the bench mark were observed and recorded. The elevation of Sta. 1 may be readily calculated from the elevation of the bench mark and the observed values. In some cases, it is more convenient to determine the elevation of the transit point from the elevation of a bench mark by direct leveling.

After the observations have been made for orienting the transit in azimuth and for determining the elevation of the first transit point, the next step is to locate the features of topography from that transit point. It is generally advan-

tageous to make the observations for each point in the same order. As previously stated, the first step should be to determine and record the stadia distance. Then, the setting should be made for determining the vertical angle, and the azimuth and the vertical angle should be observed and recorded. In the survey to which Fig. 9 applies, four points were located from Sta. 1 for the purpose of showing topographic features. In the case of the last of these four points, the middle hair of the transit was set to read 7.1 feet on the rod when the vertical angle was measured. Before the transit is moved to the next traverse corner, the vernier reading for the point on which the transit was oriented should be checked. Thus, a second sight was taken to Sta. 2 and the vernier reading was again found to be 24°12′, as shown in the notes.

45. When the work at the first traverse corner is finished, the transit is moved to the second corner. There, the height of instrument is measured and the transit is oriented by sighting to the first corner with the vernier properly set. Also, the stadia distance and the vertical angle back to the first corner are observed as a check on the values obtained for the sight from the first corner to the second. Thus, in Fig. 9, the H.I. at Sta. 2 is shown to be 4.9 feet; the reading of the vernier for the backsight to Sta. 1 was made equal to $24°12'+180° = 204°12'$; and the stadia distance and vertical angle to Sta. 1 were observed. These values should not differ much from the readings taken to Sta. 2 from Sta. 1. However, the sign of the vertical angle should obviously be different.

The next operation is to obtain complete data for the observation on the third traverse corner. The various features of topography in the vicinity of the second corner are then located in the manner described for locating the features near the first corner. In the survey for which the notes are given in Fig. 9, the second observation from Sta. 2 was made on Sta. 3, and then three points were located for the purpose of showing topography. When the vertical angle to the first of these three points was measured, the middle hair was set to 8.9 feet on the rod; and for the second point the reading of the middle

hair was 5.9 feet. The elevation of the third point was determined by reading the rod with the telescope horizontal, and therefore no vertical angle is recorded for that sight. The final operation at Sta. 2 was to check the vernier reading for the backsight to Sta. 1.

Similar operations are performed at each successive corner of the traverse.

Ver. A	Stadia Dist.	Vert. ∡	Rod Read.	Diff. Elev.	Elev.	Cor. Hor. Dist.	Object
	At	Sta. 1.	H.	I.=5.1.	Elev.	= 890.0	
24°12′	437	−3°18′		−25.1	864.9	436	Sta. 2
67 15	332	−2 11		−12.7	877.3	333	B. M. #27
173 35	625	−4 33		−49.5	840.5	622	Foot of slope
182 45	658	+1 19		+15.2	905.2	659	Top of hill
189 10	432	+2 46		+20.9	910.9	432	" " "
193 15	420	−3 44	7.1	−29.4	860.6	419	Foot of slope
24 12							Sta. 2
	At	Sta. 2.	H.	I.=4.9.	Elev.	= 864.9	
204 12	436	+3 17					Sta. 1
47 43	578	−2 14		−22.5	842.4	578	Sta. 3
155 55	446	+4 36	8.9	+31.7	896.6	444	Top of hill
152 25	427	+0 18	5.9	+1.2	866.1	428	Foot of slope
127 45	218	— —	9.3	−4.4	860.5	219	" " "
204 12							Sta. 1

Fig. 10

46. Computations for Completing Notes.—In Fig. 10 are shown the complete notes for the survey to which Fig. 9 applies. The values entered in the columns headed *Diff. Elev.* and *Cor. H or. Dist.* for each sight are determined from the values in the columns headed *Stadia Dist.* and *Vert.* ∡ for the respective sight. In this case, the stadia reduction table is used and the values of the stadia interval factor and the stadia constant are taken as 100 and 1, respectively.

In making the calculations for the sight from Sta. 1 to Sta. 2, the stadia distance R and the vertical angle a are taken as the averages of the observed values from Sta. 1 to Sta. 2 and the observed values from Sta. 2 to Sta. 1. Thus, R is 436.5 and $a = 3°17'30''$. Similarly, the averages of the values

of R and a observed at the two ends of the line between Sta. 2 and Sta. 3 would be used for computing the results for that line. However, the readings from Sta. 3 to Sta. 2 are not given in Fig. 9 and the computed values in Fig. 10 are obtained by using only the observed rod interval and vertical angle for the sight from Sta. 2 to Sta. 3.

The elevation at Sta. 1 is obtained as follows: The elevation of B.M. # 27 is known to be 877.3 and the difference in elevation between Sta. 1 and that bench mark is shown to be -12.7. Since it is required to find the elevation of Sta. 1 and the vertical angle from the station to the bench mark is negative, the station is higher than the bench mark and its elevation is $877.3+12.7=890.0$. The elevation of Sta. 2 is then computed by subtracting from 890.0 the difference in elevation between Sta. 1 and Sta. 2; the difference in elevation is subtracted because the vertical angle from Sta. 1 to Sta. 2 is negative. Thus, $890.0-25.1=864.9$. The elevation of the first point of topography is determined in a similar manner. For the second or third point, the difference in elevation is added to the elevation of Sta. 1 because the vertical angle to each of these points is positive. In the case of the last point located from Sta. 1, the elevation is determined by the method of Art. 33. The difference in elevation between the ends of the sight is 27.4 feet, and the difference in elevation between the instrument point and the stadia point is

$$5.1-27.4-7.1=-29.4$$

The required elevation is $890.0-29.4=860.6$.

The elevation of Sta. 2 is also entered in the notes on the line with the H.I. at that station. The differences in elevation between Sta. 2 and the other points that are located from that station are determined as follows:

$$4.9+35.7-8.9=+31.7$$
$$4.9+\ 2.2-5.9=+\ 1.2$$
$$4.9-\ 9.3=-4.4$$

47. Survey When Middle Hair Is Set at Foot-Mark.—When the general method in measuring each vertical angle is to set

the middle hair at a convenient foot-mark, the notes are similar to those shown in Fig. 9. However, instead of recording the H.I. and the elevation of the instrument point for each station, the elevation of the line of sight at that station is recorded. The abbreviation *E.L.S.* is used for elevation of line of sight. For example, the entry for Sta. 1 in Fig. 10 would be as follows: At Sta. 1 E.L.S. = 895.1. Also, the rod reading of the middle hair would be recorded in the column headed *Rod Read.* for every sight; and the values in the column headed *Diff. Elev.* would be the differences in elevation between the line of sight at the instrument point and the stadia point.

48. Topographic Survey With Transit Equipped With Stadia Arc.—When a transit with a stadia arc is used in a topographic survey, the field work is essentially the same as that described in Art. 44. However, the *V* and *H* scales of the stadia arc are read and those values are recorded instead of the vertical angle. Also, the modifications mentioned in Art. 47 in regard to the elevation of the line of sight, the rod readings, and the differences in elevation apply when a stadia arc is used.

INDIRECT LEVELING BY STADIA

49. Method of Procedure.—In rough country, it is frequently advantageous to run a line of levels by using a stadia transit rather than by direct leveling. Where great accuracy is not required, the method of procedure is essentially the same as that followed in direct leveling. However, the line of sight may be inclined instead of horizontal. To start the survey, a rod is held on a bench mark or other point of known elevation, the transit is set up at a convenient distance from the starting point, and the stadia distance and the vertical angle to some foot-mark on the rod are observed. Then a turning point ahead of the transit is established, the rod is held on that turning point, and the stadia distance and the vertical angle are determined for this sight. The vertical angles for the backsight and foresight should preferably be measured with the middle hair set on the same foot-mark in each case. The transit is then moved beyond the first turning point, backsight

readings of the stadia distance and the vertical angle are taken on that turning point, a second turning point is established, and foresight readings are taken on this point. These operations are repeated until the end of the line is reached.

If the vertical angles for both the backsight and the foresight from a set-up are taken with the middle hair at the same foot-mark on the rod, that rod reading may be neglected in computing the difference in elevation between the two points on which the rod was held. In case it is not convenient to set the middle hair at the same rod reading for the foresight as for the backsight, any other setting is satisfactory. However, the difference between the rod readings for the two sights must be considered in calculating the elevation of the turning point. It is advisable to make a rough sketch for each case in order to visualize the conditions.

In stadia leveling, it is very important either to eliminate the index error of the vernier on the vertical arc or to observe the error after each vertical angle is measured and to correct the vertical angle for that error.

Example 1.—A stadia transit with a stadia interval factor of 100 and a stadia constant of 1 foot was used in indirect leveling. The line of levels was started from a bench mark having an elevation of 673.5 feet. When the backsight was taken on this bench mark, the rod interval between the stadia hairs was 3.68 feet and, with the middle hair reading 5 feet, the vertical angle was found to be $-4°20'$. For the foresight on a turning point from the same set-up, the rod interval was 5.12 feet and, with the middle hair again at 5 feet, the vertical angle was $+3°46'$. Determine the elevation of the turning point.

Solution.—By the stadia reduction table, the difference in elevation between the telescope of the transit and the point at which the middle hair cut the rod held on the bench mark was

$$3.68 \times 7.53 + 0.08 = 27.8 \text{ ft.}$$

Also, the difference in elevation between the telescope and the point at which the middle hair cut the rod held on the turning point was

$$5.12 \times 6.56 + 0.06 = 33.6 \text{ ft.}$$

Since the vertical angle to the bench mark was negative, the transit was higher than the bench mark. Also, since the vertical angle to the turning point was positive, the turning point was higher than the transit. In this case, both vertical angles were measured to the 5-foot mark on the rod and the required elevation of the turning point is

$$673.5 + 27.8 + 33.6 = 734.9 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 2.—If the vertical angle to the turning point in the preceding example were taken with the middle hair reading 3 feet on the rod, what would be the elevation of the turning point?

SOLUTION.—The vertical distances of 27.8 and 33.6 ft. would be computed as in the preceding example. The difference in elevation between the bench mark and the center of the telescope is then $5+27.8=+32.8$ ft., and the difference in elevation between the center of the telescope and the turning point is $33.6-3=30.6$ ft. Hence, the elevation of the turning point is

$$673.5+32.8+30.6=736.9 \text{ ft. Ans.}$$

50. Field Notes for Indirect Leveling.—A good method of recording field notes for indirect levels run with a stadia transit in the manner described in the preceding article is shown in Fig. 11. In the first column is entered the designation for each

Rod Point	Backsight		Foresight		Elev.
	Obs.	Diff. Elev.	Obs.	Diff. Elev.	
	Rod reading for vert. angles = 5.0, $\frac{f}{i}$ = 100, f+c = 1				
B.M. #27	374 -5°10′	+33.6			877.3
T.P. 1	416 +0°44′	-5.3	289 +1°23′	+7.0	917.9
T.P. 2	177 -8°30′	+26.0	540 +4°2′(7.0)	+36.0	948.6
T.P. 3	501 +1°16′	-11.1	325 -2°52′	-16.3	958.3
T.P. 4	456 -0°37′	+4.9	492 -1°20′	-11.5	935.7
B.M. #28			278 +7°15′(4.0)	+35.9	976.5

FIG. 11

point on which the rod is held, the abbreviation *B.M.* standing for bench mark and *T.P.* for turning point. In the columns headed *Obs.*, which is the abbreviation for *Observations*, are entered the stadia distance and the vertical angle. As shown by the headings *Backsight* and *Foresight*, the observed values for the two kinds of sights are placed in separate columns. Also, under each of the headings *Backsight* and *Foresight* is provided a column headed *Diff. Elev.* for the computed difference in elevation corresponding to each stadia distance and

vertical angle. The values on one horizontal line are for the backsight and the foresight to the same point on which the rod is held, and not for the backsight and foresight from the same position of the transit. Therefore, although the columns under the heading *Backsight* are to the left of those under the heading *Foresight*, the readings for the foresight on any turning point are observed and recorded before the backsight readings on the same turning point. In the last column, headed *Elev.*, are given the elevations of the points designated in the first column. Just below the headings are recorded the following data: the usual rod reading on which the middle hair is set when the vertical angle is read, the stadia interval factor, and the stadia constant.

When the middle hair is set at a rod reading that differs from the usual value, the actual reading is entered in the notes in parenthesis after the observed vertical angle. Thus, for the foresight on the second turning point, the rod reading was 7.0 feet; and, for the foresight on B.M. # 28, the rod reading was 4.0 feet. The differences in elevation and the elevations may be computed in the manner explained in Art. 49. However, where the reading of the middle hair is not the same for the foresight as for the backsight, it is convenient to include the difference in the notes as a part of the difference in elevation for the foresight. In the case of the foresight for the second turning point, the difference between 5.0 and 7.0, or -2.0, is included in the value $+36.0$. The difference is negative if the reading of the middle hair is greater for the foresight than for the backsight; and is positive if the middle-hair reading is smaller for the foresight than for the backsight. In general, the difference in elevation has the same sign as the vertical angle for a foresight and the opposite sign for a backsight. In an occasional instance the middle hair may be set at such a reading that the line of sight and a straight line between the instrument point and the stadia point are inclined in opposite directions.

The elevation of the first T.P. is determined from the elevation of the starting point, or B.M. # 27, the difference in elevation for the backsight on that bench mark, and the difference

in elevation for the foresight on the T.P. Thus, 877.3+33.6 +7.0=917.9 feet. The values that are used for determining the elevation of the second T.P. are the elevation of the first T.P., the difference in elevation for the backsight on that T.P., and the difference in elevation for the foresight on the second T.P. The required elevation is 917.9−5.3+36.0=948.6 feet. A similar procedure is followed for calculating each of the other elevations given in the last column.

51. Method of Obtaining Greater Precision.—Where a higher degree of accuracy is desired, the transit should be set up at each turning point instead of between turning points, and the following method should be employed. At each set-up the first step is to measure the H.I. to the nearest hundredth of a foot. Then the stadia distances and the vertical angles for the backsight and the foresight are observed. Whenever possible, the vertical angles from a set-up should be read with the middle hair cutting the rod at the H.I. of the transit. If the index error has not been eliminated, it is also observed for each sight.

The difference in elevation between each two successive points should be found by computing the difference in elevation for the foresight and also the difference in elevation for the backsight, and then taking the mean of these two results.

52. Field Notes for Accurate Method.—In Fig. 12 is shown a form of field notes that is suitable for stadia leveling when the method described in Art. 51 is employed. For each set-up, the designation of the point and the H.I. are recorded first. In general, the designations of the points for the backsight and the foresight are entered in the column headed *Station*. At the first set-up, however, there is only a foresight; and, at the last set-up, there is only a backsight. The stadia distance and the vertical angle for each sight are recorded in the columns having the headings *Stadia Dist.* and *Vert.* $\measuredangle$, respectively. The difference in elevation is computed for each sight independently, and the values are placed in the column headed *Diff. Elev.* The elevations in the column headed *Elev.* are determined in the following manner.

Each value entered in the column headed *Diff. Elev.* is the difference in elevation between the point over which the transit was set and the point on which the rod was held. However, since there is a foresight and a backsight for each line, the

Station	Stadia Dist.	Vert. ∠	Diff. Elev.	Elev.
⊼ at B.M. #65	H.I. = 4.80	Elev. = 649.12	$\frac{f}{i} = 100$	f + c = 1
T.P. 1	317	+1°41′	+9.35	658.47
⊼ at T.P. 1	H.I. = 4.96			
B.M. #65	317	−1°41′	+9.35	
T.P. 2	454	+3°24′	+26.94	685.45
⊼ at T.P. 2	H.I. = 4.73			
T.P. 1	453	−3°25′	+27.01	
T.P. 3	511	−2°20′	−20.84	664.49
⊼ at T.P. 3	H.I. = 4.70			
T.P. 2	513	+2°21′	−21.07	
T.P. 4	726	+0°30′	+6.33	670.70
⊼ at T.P. 4	H.I. = 4.84			
T.P. 3	724	−0°29′	+6.09	
B.M. #66	190	+5°57′	+19.69	690.39
⊼ at B.M. #66	H.I. = 5.03			
T.P. 4	190	−5°57′	+19.69	

FIG. 12

difference in elevation between two successive points, as used in determining elevations, is taken as the average of the values for the foresight and the backsight between the points. The elevation of B.M. # 65 is known to be 649.12 feet. Since the difference in elevation between that bench mark and the first

turning point is found to be 9.35 feet for both the foresight from the bench mark and the backsight from the turning point, the elevation of T.P. 1 is assumed to be $649.12+9.35=658.47$ feet. Also, the difference between the elevations of T.P. 1 and T.P. 2 is taken as $\frac{26.94+27.01}{2}=26.98$ feet, and the elevation of T.P. 2 is recorded as $658.47+26.98=685.45$ feet. The elevation of each of the other points is determined in a similar manner.

53. Indirect Leveling When Stadia Arc Is Used.—When indirect leveling is done with a transit that is equipped with a stadia arc, the procedure in making the observations is essentially similar to that described when vertical angles are measured, but no attempt is made to set the middle hair at a

Stadia Arc Reading		Rod Interval	Product	Rod Read.	Diff. Elev.	Elev.	Station
B.S.	F.S.						
						877.3	B.M.#27
41		3.74	+33.7	+4.9	+38.6	915.9	E.L.S.
	52	2.89	+5.8	−3.8	+2.0	917.9	T.P.1
51		4.16	−4.2	+3.9	−0.3	917.6	E.L.S.
	57	5.40	+37.8	−6.8	+31.0	948.6	T.P.2
35		1.77	+26.6	+4.4	+31.0	979.6	E.L.S.
	45	3.25	−16.3	−5.0	−21.3	958.3	T.P.3
52		5.01	−10.0	+3.9	−6.1	952.2	E.L.S.
	48	4.92	−9.8	−6.7	−16.5	935.7	T.P.4
50		4.56		+9.9	+9.9	945.6	E.L.S.
	63	2.78	+36.1	−5.2	+30.9	976.5	B.M.#28

FIG. 13

predetermined rod reading. A good form of notes is as shown in Fig. 13. The abbreviations *B.S.* and *F.S.* in the headings of the first two columns stand for backsight and foresight, respectively, and the abbreviation *E.L.S.* in the last column stands for elevation of line of sight.

The values in the column headed *Product* are obtained by multiplying the rod interval by the difference between the respective stadia-arc reading and 50. For a backsight, this

product is considered positive when the stadia-arc reading is less than 50, because the line of sight from the rod toward the telescope is then inclined upward; and the product is considered negative when the stadia-arc reading is more than 50. For a foresight, the reverse is true, and the product has the same sign as the result obtained by subtracting the stadia-arc reading from 50. Also, the rod reading is always positive for a backsight and negative for a foresight. Each value in the column headed *Diff. Elev.* is the algebraic sum of the values in the columns headed *Product* and *Rod Read.* The difference in elevation is, in every case, added algebraically to the elevation on the preceding line to obtain the elevation on the same line with the difference. Thus, $877.3+38.6=915.9$; $915.9+2.0=917.9$; and $917.9-0.3=917.6$.

PLANE-TABLE SURVEYING

OUTFIT FOR PLANE-TABLE WORK

DESCRIPTION OF EQUIPMENT

54. Plane-Table Map.—The instrument known as the plane table is used extensively in the preparation of maps that show the topography of an area. When the plane table is used, the various points that are located with its aid and the topographic features are plotted directly on the map in the field, and no written notes need be kept. The topographer has the opportunity to compare his map with the ground and to see whether or not the various features are properly shown on the map.

55. Essential Parts of Plane-Table Outfit.—As shown in Fig. 14, a plane-table outfit consists essentially of a drawing board which is mounted on a tripod in such a manner that the board can be leveled and revolved about a central vertical axis without disturbing the tripod; and also of an instrument, known as an alidade, for sighting in any desired direction and transferring the direction of the line of sight to drawing paper that is held in position on the board. Strictly speaking, the term plane table means only the board and its tripod. How-

ever, it is customary to apply the term to include also the alidade, and this broader meaning will be employed in the following explanations.

A tripod for a plane table is of the same general design as one for a transit, but the legs are heavier and shorter so that the table can be set up firmly at a height which is convenient for drawing. Three types of plane tables are in common use: namely, the *Coast-Survey type*, the *Johnson type*, and the *traverse type*.

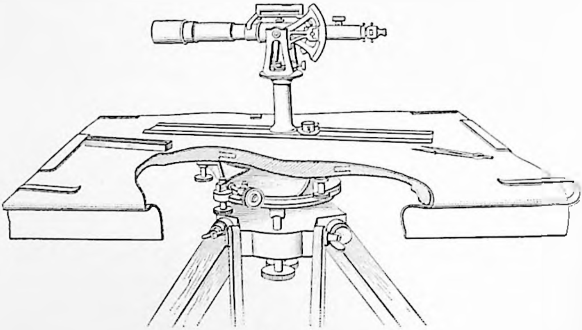


FIG. 14

56. Drawing Board and Movement.—The drawing board of a plane-table outfit is made of well-seasoned pine. To overcome the tendency of the board to warp, a strip that runs at right angles to the grain in the main part of the board is dovetailed into the main part at each end. In the Coast-Survey table and in the Johnson table, the board is either 24 in. × 31 in. or 18 in. × 24 in. In a traverse table, the board is 15 inches square. The difference between the Coast-Survey and Johnson tables is in the movement, or the device by which the board is leveled and rotated on the vertical axis.

The construction of the Coast-Survey movement is illustrated in Fig. 14, where a portion of the board is shown broken away so as to expose the movement. This movement is a

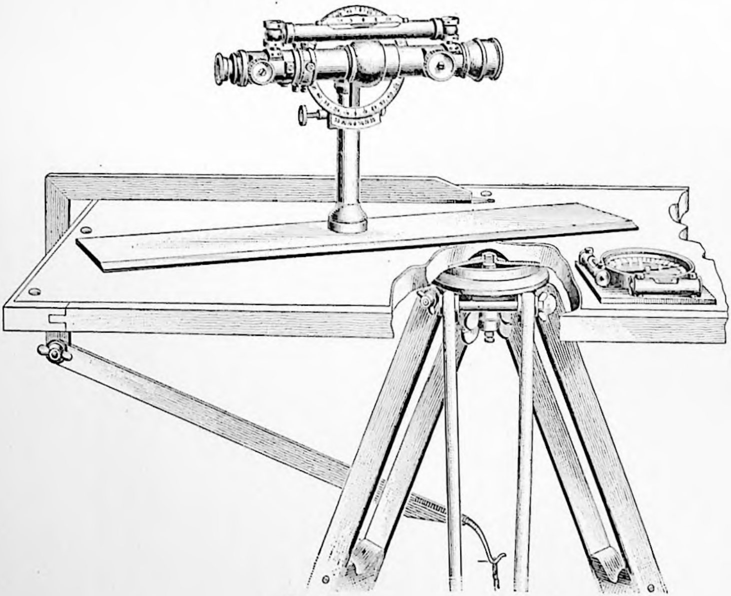


FIG. 15

casting that is provided with three leveling screws for leveling the board and also with a clamp and tangent screw for controlling the rotation of the board on the vertical axis.

The Johnson movement is illustrated in Fig. 15. A circular brass plate with a threaded opening is embedded securely in the under side of the board and the board is attached to the tripod by screwing this plate onto a threaded projection on top of the tripod head. The tripod head consists of two castings which form a ball-and-socket joint. At the base of the joint are two nuts or clamps, one above the other. To level the board, both clamps are loosened and the board is adjusted by hand until it is horizontal. It is fixed in that plane by tightening the upper clamp. When the lower clamp is loose, the board can be rotated on the central axis. The board is held in any desired position by tightening the lower clamp. The Johnson table weighs less than the Coast-Survey table, but it cannot be leveled so accurately.

In the case of a traverse table, the board can be rotated in azimuth and can be clamped in any desired orientation, but there is no leveling device. The board is brought approximately to a horizontal position by moving the tripod legs. This table is convenient to carry, as it is small and light in weight, but it is not suitable for accurate work.

57. Alidade.—Plane-table alidades are of two general types. In one type, known as a *peep-sight alidade*, two sight vanes like those on a surveyor's compass are placed at the ends of a ruler. The sights may be either fixed in a vertical position or hinged so that they can be set vertical when the table is in use and can be folded down flat on the ruler at other times. The ruler is generally a brass straightedge that is 6 to 10 inches long. One edge is beveled and graduated for use as a measuring scale.

Alidades of the other type are known as *telescopic alidades*. Three forms of telescopic alidades are illustrated in Figs. 14, 15, and 16. The telescope is mounted on a vertical column which is attached to a metal ruler, and it can be rotated in a vertical plane that passes either through one edge of the ruler or through

a parallel line. However, in the case of alidades like those shown in Figs. 14 and 15, the telescope is rigidly attached to the transverse axis on which it rotates, as is the telescope of a transit; whereas, in the case of the alidade shown in Fig. 16, the telescope is supported in a cylindrical sleeve which is rigidly attached to the transverse axis, and the telescope may be turned in the sleeve about its line of collimation, as may the telescope of a wye level. Every telescopic alidade is provided with the usual vertical and horizontal cross-hairs for establishing directions and elevations and also with stadia hairs for determining distances by the stadia method.

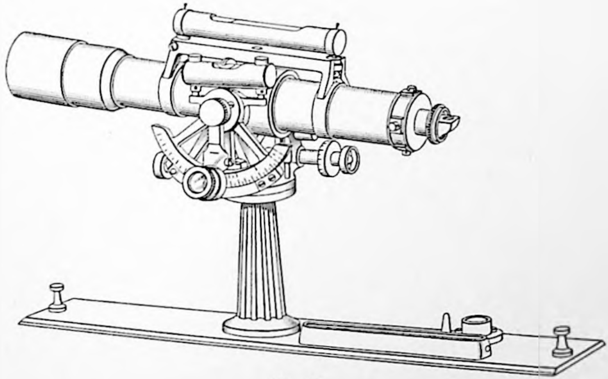


FIG. 16

For the measurement of vertical angles, the telescope is equipped with a vertical arc and with a clamp and tangent screw. Most telescopic alidades may be leveled by means of a removable spirit level, known as a *striding level*, which is supported on top of the telescope in the manner illustrated in Figs. 14 and 16. Some alidades have a Beaman stadia arc in addition to the vertical arc. The vernier for the vertical arc is held in place by screws or is mounted so that it can be moved independently of the telescope by means of a tangent screw.

If the vernier is movable, the arm on which it is mounted also carries an auxiliary spirit level.

58. The ruler on which a telescopic alidade is mounted is a metal straightedge that is about 18 inches long and 3 inches wide and is beveled on one edge. Such a ruler is usually equipped with a circular bubble which is centered when the drawing board is level. The rulers illustrated in Figs. 14 and 16 have circular bubbles. In some cases, the ruler is also provided with a compass box containing a magnetic needle. The long narrow projection with oval ends on the top of the ruler shown in Fig. 16 is a compass box. The rulers of some of the older forms of alidades, such as that shown in Fig. 15, have neither a circular bubble nor a compass box.

59. Paper for Plane-Table Work.—When a plane-table survey is made, the map is plotted on drawing paper that is attached to the drawing board of the plane-table outfit. The paper is sometimes used in rolls, the portions not in use being rolled under the ends or sides of the board. More often the paper is prepared in the form of sheets about the size of the board. It is held on the board by screws, by heavy brass clamps, or by adhesive tape.

The paper should be of good quality, as the expansion and contraction due to atmospheric changes is very large in the poorer grades of paper. It may be seasoned by being exposed for several days alternately to a very damp and to a very dry atmosphere. This seasoning considerably lessens the effect of atmospheric changes. If a high degree of accuracy is desired, the map is usually drawn on paper of the best grade that is mounted either on cloth or on thin sheets of aluminum. Celluloid sheets are sometimes used where the map is likely to become moistened by rain, dew, or water dropping from the trees.

60. Accessories for Plane-Table Work.—It is not ordinarily necessary to set up the plane table so that a certain point on the paper will be exactly over a given point on the ground. Therefore, the table is generally set up without the aid of a plumb-bob. However, if desired, a plumb-bob can be sus-

pended underneath the board either from the hand of the topographer or from a special device, known as a *plumbing arm,* one type of which is shown in Fig. 15.

When the ruler of the alidade is equipped with a circular bubble but not with a compass box, a separate compass box is often used. A separate compass box is shown in Fig. 14 in front of and near the left end of the ruler of the alidade. In the case of a traverse table, a compass box is placed in a trough along one edge of the board. When the alidade has neither a circular bubble nor a compass box, the drawing board can be leveled and the direction of the magnetic meridian can be determined by means of a device known as a *declinator,* which may be placed on the board. A declinator is illustrated at the right in Fig. 15. It consists of a square metal plate that is equipped with a compass box and two bubble tubes which are at right angles to each other. Two edges of the plate are parallel to the line through the zero marks of the compass circle so that, when the compass needle reads zero, those edges give the direction of the magnetic meridian.

In addition to the plane table, it is necessary to have such accessories as stadia rods, a scale, triangles, a stadia slide-rule, pins, pencils, and an eraser. As an aid in keeping the map clean, the drawing paper is generally covered with a piece of wrapping paper, and only the part on which work is being done is exposed. A waterproof cover should be provided to protect the map from sudden showers.

FUNDAMENTAL OPERATIONS WITH PLANE TABLE

61. Setting up Plane Table.—To be theoretically correct, the plane table should be set up so that the plotted position of the station over which the instrument is placed is directly above the station on the ground. However, the table is usually considered to be set up correctly if the center of the board is nearly over the station on the ground, because the scale of the map is generally so small that an error of 1 foot would not show on the map.

In order to set up the plane table, the legs of the tripod are first spread so that the board will be about waist high; and the

legs are then planted firmly in the ground with the tripod head approximately level and the board satisfactorily centered over the station on the ground. In case it is necessary to move the table so as to bring a certain point on the map more nearly over the station on the ground, one or two of the tripod legs may be pressed further into the ground, or else the tripod may be lifted and moved as a unit without changing the relative inclinations of the legs. When the table has been properly centered, the board is leveled by means of the attachment that is provided for the purpose. If the table is of the Johnson type, the inclination of the board is changed by pressing on the sides with the hands. With a table of the Coast-Survey type, the board is leveled by means of the leveling screws. Either the circular bubble on the alidade or the bubbles on a declinator may be used to show when the board is level.

It is not customary to attempt to have the board precisely level for each observation. Since the location of the various points from each set-up requires shifting of the alidade on the board, and since few tables are so rigid that the board would remain exactly level as the alidade is moved about, the board need not be leveled with extreme care. However, it is convenient to have the board nearly level.

62. Establishing Direction of Line Between Station Occupied by Plane Table and Another Point.—To take a sight to a certain point from a plane table, the alidade is first laid on the board with the working edge of the ruler passing through the point on the map representing the position of either the station occupied by the table or the point to which the sight is taken. The observer then looks through the telescope of the alidade and changes the direction of the line of sight until it passes through the point to which the sight is taken. The direction of the line of sight of the alidade may be changed either by turning the board on its vertical axis or by shifting the position of the alidade on the board.

When the station occupied by the plane table has been previously plotted and the table is to be oriented by sighting to another point that has already been plotted, the required direc-

tion of the line of sight is established by turning the board. This process is often called backsighting. After the plane table has been oriented, the required direction of the line of sight to a point that has not yet been plotted is established by shifting the alidade on the board. This process is usually known as foresighting..

When the alidade is shifted, care must be taken to have the edge of the ruler pass through the plotted point. To facilitate sighting when the alidade is to be shifted, a fine plotting needle may be stuck into the board vertically at the plotted point, and the edge of the ruler may then be kept in contact with the needle. The direction of the line of sight for any position of the alidade is indicated by the edge of the ruler. Hence, the direction of the line through the station occupied by the plane table and the point to which the sight is taken may be plotted by drawing a line along the edge of the ruler when the line of sight is properly directed.

63. Orienting Plane Table.—When two or more points have been plotted on the map, orienting the plane table consists in clamping the board in such a position that the line between any two plotted points on the map is parallel to the line between the corresponding points on the ground. The method of orienting the plane table at a particular station depends on the conditions. When the station occupied by the table has been previously plotted on the map, either of the following two methods can be applied.

Where a high degree of accuracy is unnecessary, the table can be oriented with the compass needle. If the ruler of the alidade is equipped with a compass box, an edge of the ruler is placed along a line on the map that represents the magnetic meridian. The board is then turned on its vertical axis until the reading of the compass needle is 0° and is clamped in that position. In case a declinator or a separate compass box is used for orienting the table, an edge of the declinator or box is placed along the line representing the meridian and the board is turned so that the needle reads 0°. This method can be used even if no other plotted station is visible.

The second method of orienting the table at a previously plotted station can be used only if a backsight can be taken to some other station that has already been plotted. An edge of the alidade ruler is placed along the line on the map joining the plotted positions of the station occupied and the other station. The table is then turned on the vertical axis until the line of sight of the alidade passes through the other station, and the board is clamped in this position.

The plane table can also be oriented at a station whose position has not been previously plotted on the map. The position of the station on the map is then determined while the plane table is being oriented. Methods for solving this problem will be explained later.

SURVEYING WITH PLANE TABLE

LOCATING POINTS FROM PLANE TABLE

64. Methods of Locating Points.—There are two general methods of locating a point from the plane table. In one method, the direction and length of the line from the table to the point are determined. In the other method, called the *method of intersection*, the directions to the point from two positions of the plane table are established, and the point is located at the intersection of those two directions. For convenience in the following explanations, points on the ground will be designated by capital letters, and the corresponding points on the map will be designated by corresponding small letters. Thus, the plotted position of a point A on the ground will be marked a.

65. Location by Direction and Distance.—The method of locating a point by its direction and distance from the plane table is similar to that described in Art. 24 for a transit. In Fig. 17, points A, B, C, D, and E are the corners on the ground of a small tract of land, and O is the station occupied by the plane table. The rectangle shown near the center of the illustration represents the plane table, the point o is the plotted position of the plane-table station, and the polygon *abcde* represents the tract on the map. The letter O is not shown

in the illustration, because the plotted position *o* of the plane-table station is assumed to be directly over that station on the ground. The size of the plane table in comparison with the area of the tract on the ground is here exaggerated in order that the tract may be shown clearly on the map. After the position *o* of the plane-table station *O* has been plotted on the map and the table has been set up and properly oriented, the points *a*, *b*, *c*, *d*, and *e* are plotted in the following manner:

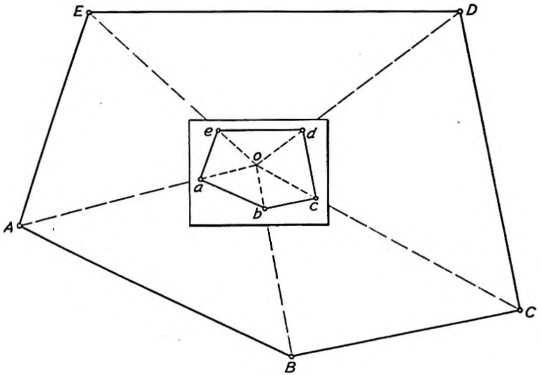


FIG. 17

The alidade is sighted to point *A* by shifting it on the board with the straightedge passing through the plotted point *o*, and a straight line of unlimited length is drawn along the straightedge. This line indicates the direction of the line on the ground from the plane-table station to *A*. Then the distance from *O* to *A* is measured by stadia or by taping, and this distance is laid off to scale on the map from *o* along the plotted line directed toward *A*. The other end of the scaled distance is the desired point *a*.

To locate the point *b*, the alidade is shifted on the board so that the straightedge passes through the point *o* and the line of sight is directed to *B*; a straight line is drawn from *o* along

the straightedge; the distance from O to B is determined; and this distance is laid off to scale along the plotted line. The other points c, d, and e are located in a similar manner by making observations to points C, D, and E, respectively.

66. Method of Intersection.—To locate a point by the method of intersection, sights are taken from two positions of the plane table, but it is not necessary to measure the distance from either position of the table to the point. This method is, therefore, particularly suitable for the location of distant or

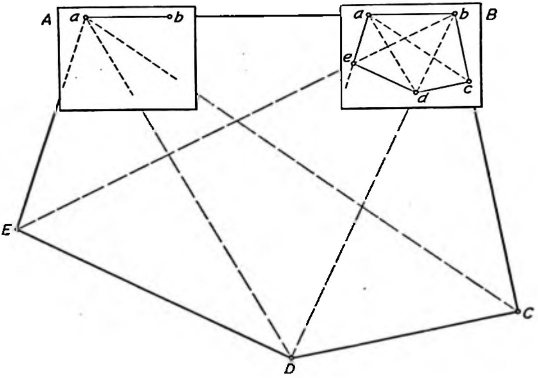


FIG. 18

inaccessible points where the measurement of distances is difficult or impossible. In Fig. 18 is indicated the method of locating the corners of a five-sided tract of land $ABCDE$ by determining the length of one side AB and setting up the plane table at the extremities A and B of that side. With the points a and b plotted on the plane-table paper, the procedure is as follows:

The table is set up at A and is oriented by setting the alidade along the line ab on the map, rotating the board until the line of sight passes through B, and clamping the board in that

position. Then, the line of sight is directed to points C, D, and E in succession by shifting the alidade, and for each sight a line of unlimited length is drawn along the straightedge from the point a. These lines are indicated by short dashes in Fig. 18.

The plane table is now moved to B, where it is oriented by setting the alidade along the line ba, rotating the board so that the line of sight passes through point A, and clamping the board. Sights to C, D, and E are then taken from B, and the direction of each sight is plotted by drawing a line from b along the straightedge. Obviously, the points c, d, and e are located at the intersections of these lines with the lines from a to the respective points.

PLOTTING POSITION OF PLANE TABLE

67. General Conditions.—In a plane-table survey it is often advantageous to set the table over a point that has not been previously located on the map. There are several methods of plotting this point and orienting the table, provided two or three points that have already been plotted can be seen from the table.

68. Location by Distances from Two Plotted Points.—When two points that have already been plotted are near the table and accessible, the problem of locating the plane-table station on the map is usually solved in the following manner: The distance from the table to each of the two reference points is measured, and arcs are described on the map with the reference points as centers and the respective distances from the plane table as radii. The intersection of the two arcs is evidently the required point.

The method is illustrated in Fig. 19. Here, A and B are two points whose positions have been plotted at a and b, and C is the station occupied by the plane table; and it is desired to locate the point c on the map. The distances CA and CB are measured either by stadia or by chaining. Then an arc is described with a as a center and the distance CA, to the scale of the map, as a radius; and a second arc is described with b

as a center and the distance CB, to scale, as a radius. The required point c is at the intersection of these arcs.

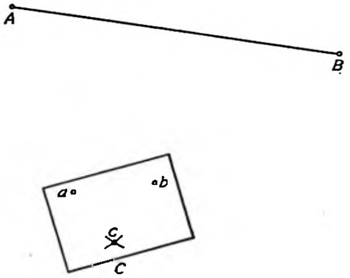


FIG. 19

To orient the table for locating additional points, the straightedge is set along the line ca or cb, the line of sight is directed to A or B—as the case may be—by rotating the board on its axis, and the table is clamped in that position.

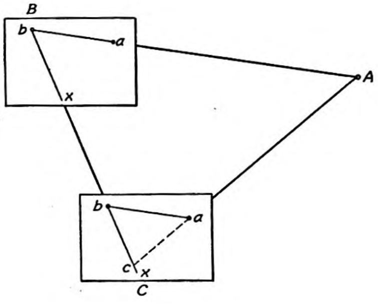


FIG. 20

69. Location by Resection.—Another problem that sometimes occurs in plane-table work is illustrated in Fig. 20. In this case, A and B are two points whose positions have been plotted at a and b, and C is the station occupied. While the

plane table was set up at B, a sight was taken toward C and the line bx was drawn to show the direction of the line from B to C; but the distance from B to C was not determined. For instance, B and C may be two points on the center line of a straight stretch of a highway or a railroad and it may not be convenient to decide on the exact location of point C before leaving station B. In other cases, measurement of the distance BC may be impracticable because the points are very far apart or are separated by a body of water. The direction of the line BC could then be established, whereas the distance BC would not be known.

To locate the plotted position c of the station C, the plane table is first oriented at C by setting the straightedge along the line xb and sighting to B. Then, with the board clamped in this position and the edge of the ruler pivoted at a, the alidade is shifted on the board until the line of sight is directed to A. The final step is to draw a line along the straightedge so as to intersect the line bx. Obviously, the required point c is at the intersection. This process is known as resection. A line that is drawn through the previously plotted position of a point to which a sight is taken, in order to locate the station occupied by the plane table, is called a resection line. Thus, ax is a resection line.

70. Indirect Methods of Locating Station Occupied.—When the station occupied by the plane table has not been sighted to before, and its distances from two previously plotted points cannot be conveniently obtained, its position on the map is determined by the solution of one of two problems that are known, respectively, as the three-point problem and the two-point problem. The three-point problem occurs when three plotted points are visible from the station occupied, and the two-point problem when only two such points are visible. Whenever possible, the station to be occupied should be chosen so that three points already located on the map can be seen from it. If only two such points are available, the plane table must be set up at an auxiliary point before the regular station can be located.

Several methods have been developed for solving the three-point problem with the plane table, but only two of them are described here. One of these is a mechanical method known as the tracing-cloth method, and the other is a trial method known as the Lehmann method. A geometrical method known as Bessel's method was formerly in common use because it is theoretically exact, whereas the other methods are approximate; but the necessary constructions have been found so complicated for field work that the method is seldom employed in present practice.

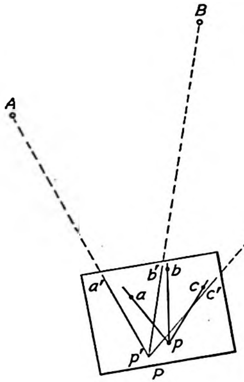


Fig. 21

71. Solution of Three-Point Problem by Tracing-Cloth Method.—The mechanical solution of the three-point problem by the use of a piece of tracing cloth or tracing paper is shown in Fig. 21. Here, A, B, and C are three points which are plotted on the map at a, b, and c, respectively, and are visible from the plane table at P. The problem consists in plotting the point p on the map so that its position with respect to the points a, b, and c will correspond to the position of P with respect to A, B, and C. The point P on the ground is assumed to be directly below the point p on the map.

For convenience the board is approximately oriented by eye or by the magnetic needle. A piece of tracing cloth or paper, that is large enough to cover the three plotted points and the estimated location of the station occupied, is fastened to the board over the plane-table paper. A point p' on the tracing cloth is so chosen that its position with reference to the points a, b, and c will be approximately the same as the position of Sta. P with reference to the points A, B, and C. With the edge of the alidade pivoted at p', the alidade is directed successively to A, B, and C, and the lines of sight are plotted on the tracing cloth, as shown at $p'a'$, $p'b'$, and $p'c'$. The tracing cloth is then unfastened and is shifted on the drawing paper to a position in which the lines plotted on the cloth pass through the points a, b, and c on the paper. In this position the point p' at the intersection of the lines on the cloth is over the required position of p on the map. The point p can be located on the map paper by pricking a small hole through the point p' on the cloth by means of a fine needle. Care should be exercised to keep the tracing cloth free from wrinkles and not to stretch it in the process of fastening it over the drawing paper.

The tracing cloth is now removed and the plane table is oriented in the following manner: The edge of the alidade is placed in contact with the point p and one of the previously plotted points, such as a, and the table is turned in azimuth until the line of sight is directed to the corresponding point A on the ground. The position of p can be checked by sighting to the points B and C with the edge of the alidade in contact, respectively, with the points b and c, and plotting the lines of sight. If the work has been done accurately, these lines will intersect at p.

If the point P should happen to be on the circumference of a circle through the points A, B, and C, the point p will be on the circumference through a, b, and c. The position of p will then be indeterminate, because an infinite number of positions of the tracing cloth could be found where the lines $p'a'$, $p'b'$, and $p'c'$ would pass through a, b, and c. However, it is highly improbable that such a point will be chosen.

72. Lehmann's, or the Coast-Survey, Method.—In Lehmann's solution of the three-point problem, the location of the required point is found by trial. When the plane table has been set up, it is oriented as closely as possible by eye or by the compass, and a resection line is drawn through each of the three plotted points that are to be used in the particular

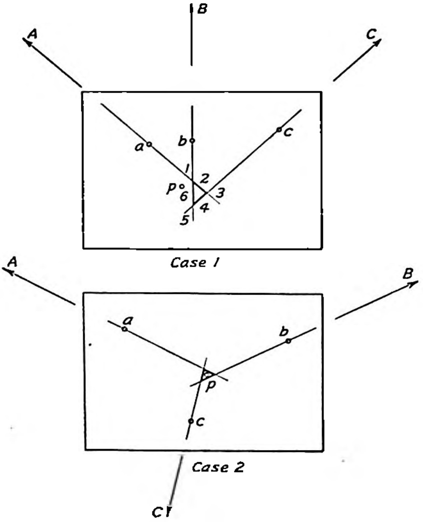


Fig. 22

case. Since the table is not properly oriented, these three resection lines will not intersect at one point, but will form a triangle of error, the size of which will depend on the error of orientation. In either of the two cases illustrated in Fig. 22, A, B, and C represent the three reference points and a, b, and c are their respective plotted positions; and the resection lines through a, b, and c are assumed to form a triangle near the required point p, as shown.

The position of the point sought can be estimated very closely by keeping in mind the following two fundamental principles: (1) The point sought is either to the right of each of the three resection lines forming the triangle of error or to the left of each of those lines; in applying this principle, the observer must always face toward the reference point toward which the particular resection line is drawn. (2) The perpendicular distances from the resection lines to the point sought are proportional to the distances from the station occupied to the respective reference points on the ground. Thus, in the first case illustrated in Fig. 22, the plotted position p of the table must be either in sector 3, which is the only one that is to the right of each of the three resection lines; or in sector 6, which is the only one to the left of all three lines. Since Sta. P is nearer to point B than to point C, the point p must be nearer to the resection line through b than to the line through c, and therefore must lie in sector 6, as shown. In the second case illustrated in Fig. 22, any point that is to the left of all three resection lines is inside the triangle of error. For this condition, it is impossible for any point to lie to the right of all three resection lines. The position of the point p in sector 6 in case 1 or within the triangle of error in case 2 is established by applying the principle in regard to proportionate distances. In case 1, the point is about equidistant from the lines through a and b, and is somewhat farther from the line through c. In case 2, the point p is nearest the line through c, and is about equidistant from the other lines.

When the position of the point sought has been plotted tentatively, the alidade is placed so that the edge of the ruler passes through the plotted positions of the station occupied and the most distant of the three reference points that were used. The table is then oriented by sighting to that reference point, and new resection lines are drawn by sighting to the other two reference points. If the estimated position of the point sought is not correct, a second, but smaller, triangle of error will be obtained. A new position of the point sought is then selected by applying the two fundamental principles to the new resection lines. If necessary, the operations are

again repeated. Practice in the use of this method will enable the topographer to develop greater accuracy in making the first approximate setting of the plane table, and one trial will be sufficient in most cases.

73. The Two-Point Problem.—It is sometimes necessary to locate and orient the plane table when only two known points are visible. By setting up the table at an auxiliary point before it is set up at the regular station, the board can be oriented at the regular station and the plotted position of that station can be determined. Whenever conditions permit, the point selected for the auxiliary set-up should be on line with the two known points, because the work is thus simplified, but any other position may be chosen for the auxiliary point.

If the auxiliary point is on line with the previously plotted reference points, the solution of the two-point problem is as follows: The plane table is oriented at the auxiliary point by placing the ruler along the line joining the two known points on the map and sighting to the more distant of those points. With the alidade in any convenient position on the sheet, a pointing is made toward the regular station to be occupied by the table, and a line is drawn along the edge of the ruler. A flag is left at the auxiliary point, and the table is moved to the point whose position is sought. It is oriented at this point by placing the alidade along the line that was drawn at the auxiliary station and sighting to the flag. The position of the point sought is then found by resecting on the two known points.

74. The solution of the two-point problem, when it is impracticable to set the plane table on line with the two reference points, is shown in Fig. 23. Here, A and B, whose positions are plotted at a and b, are the reference points, and C is the station to be occupied by the plane table. The table is first set up at any auxiliary point D, and is oriented as closely as possible by eye or by the compass. The point d' is located at the intersection of resection lines through the two known points A and B. This is not the true location of d, because

the board was not correctly oriented. The alidade is next directed to the required point C, the line of sight is drawn, and the distance $d'c'$ along that line is laid off to represent to scale the estimated distance from D to C.

The table is moved to C and is oriented by placing the edge of the alidade along the line $c'd'$ and backsighting to D. With the alidade pivoted on c', a sight is then taken to A, and the point a' is located at the intersection of the edge of the alidade with the line through d' and a. Similarly, a sight

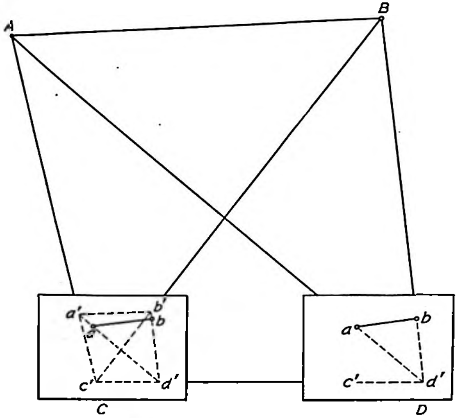


FIG. 23

is taken to B, and the point b' is located at the intersection of the straightedge with the line through d' and b. The quadrilateral $a'b'd'c'$ has the same shape as that formed by the four points on the ground, but is erroneous in size and position. However, the line $a'b'$ is parallel with the line AB on the ground, and so the error in orientation is indicated by the angle between ab and $a'b'$.

A convenient method of correcting the orientation of the table is to place the edge of the alidade along the line $a'b'$ and to note some distinctive and distant object that is bisected

by the vertical cross-hair of the telescope. The alidade is then moved to the line *ab* and the table is turned until the same object is again bisected. The table is then properly oriented and the correct position of *c* is found by resecting on *A* and *B*.

CONTROL FOR PLANE-TABLE SURVEYS

75. Horizontal Control.—There are two general methods of conducting a plane-table survey for locating topography. In one method, all the work is done with the plane table, each new plane-table station usually being plotted by the direction and distance from a previously occupied station or by intersection from two stations. The main objection to this method is that any error in the location of a station affects the locations of all points beyond the one at which the error occurred. In some surveys, few field checks are available until the traverse is closed. If the closing error is considerable, the adjustment of the topography, which has been taken from different stations, may be rather difficult. For these reasons, the survey is conducted entirely with the plane table only when the area is small or when numerous field checks are provided as the work progresses.

A survey based on triangulation control or on a transit-tape traverse, or even on a transit-stadia traverse, will be more satisfactory. When this control is available, the positions of the control points are plotted on the plane-table sheet or sheets before the field work is begun. The plane-table survey may then begin at one of the stations, and the board may be oriented by sighting on an adjacent station; or the plane table may be set up at any convenient point and its location determined by the three-point method.

In case the control is a triangulation system, it is probable that additional plane-table stations will be needed to fill in the required topography. When such intermediate stations are located by running traverses with the plane table, the traverses usually begin and end on triangulation stations. As these traverses are generally short, the comparatively small closing errors can be adjusted more easily than can the closing errors when the entire survey is conducted with the plane table.

In order that a triangulation station can be seen from the plane-table stations in various directions, a signal is usually erected at the triangulation station after the point has been occupied by the transit. The signal generally consists of a straight pole that is braced so as to stand in a vertical position and has a piece of cloth attached to the top to make it more easily discernible.

When the entire survey must be made without control of higher precision, time will often be saved by running a closed traverse with the plane table before any detail is located. After the traverse has been adjusted on the plane-table sheet, the stations are occupied a second time to locate the topography.

76. Vertical Control.—The ideal vertical control for a plane-table survey is afforded when the elevations of all plane-table stations are known in advance. These elevations may be determined by direct leveling; but, when either a transit traverse or a plane-table traverse is run in advance of the topography, elevations are often obtained by means of vertical angles and stadia distances. These angles and distances should be observed from both ends of the line. Any reasonable closing error in the differences of elevation is distributed before the field work with the plane table is begun.

In case the plane-table stations have not been established in advance of the leveling, bench marks should be set at frequent intervals. If this is done, plane-table elevations can be determined and checked by sighting on the bench marks, and errors in elevation can thus be kept to a minimum.

Where the plane table is located by the three-point method, the elevation of the new station is calculated from the elevations of the three known points. Vertical angles are measured to definite points on the signals that are at known distances above the stations. The differences of elevation are computed from these vertical angles and from horizontal distances which are scaled from the plane-table sheet. As the plane table will seldom be level for all positions of the alidade, the index error for each vertical angle should be either eliminated, if the vernier of the vertical arc is movable, or determined by

reading the vertical circle with the bubble on the telescope centered. If the telescope can be rotated in its bearings, instrumental errors caused by the horizontal cross-hair being out of adjustment should be eliminated by rotating the telescope through an angle of 180° in its bearings and taking a second observation of each vertical angle with the telescope in this position. A correction for curvature and refraction should be subtracted from each computed difference of elevation. This correction, in feet, can be taken as $0.02D^2$, where D is the horizontal distance, in thousands of feet. The adopted elevation of the alidade is taken as the mean of the three values computed from the three reference points. The elevation of the plane-table station is obtained by subtracting the H.I. of the alidade—which is the height of the alidade above the station—from the elevation of the alidade. The method is illustrated by the following example:

With the plane table set up over a point designated as Sta. *P*, at which the H.I. is 4.1 feet, observations are taken on three plotted points *A*, *B*, and *C*. The data are shown in the first six columns of the accompanying tabulation. Thus, for the

COMPUTATIONS FOR ELEVATION OF PLANE TABLE

Pointing	Vert. Angle	Index Error	Scaled Dist.	Elev. Sta.	Ht. of Pt. Above Sta.	Diff. Elev.	Curv. & Ref.	Elev. Alidade
A	+1°14′	+02′	6,230	876.3	+16.6	−130.5	−0.8	761.6
B	−0 24	−03	10,740	674.8	+24.2	+ 65.5	−2.3	762.2
C	−0 43	00	7,620	648.9	+17.8	+ 95.3	−1.2	760.8
							Mean	= 761.5
							H.I.	= −4.1
							Elev. Sta. *P*	= 757.4

sight to point *A*, the vertical-angle reading was +1°14′ and the index error was +2′. Also, the horizontal distance from *P* to *A* was scaled from the map as 6,230 feet, the elevation ot point *A* was known to be 876.3 feet, and the vertical angle was

measured to a point on the signal 16.6 feet above point A. The corrected vertical angle is $1°14'-2'=1°12'$ and the difference in elevation between the alidade at P and the point 16.6 feet above A is 6,230 tan $1°12'=130.5$ feet. Since the vertical angle from Sta. P to point A is positive, P is lower than A and -130.5 is entered in the seventh column of the tabulation. The correction for curvature and refraction is $0.02 \times 6.23^2 = 0.8$ foot. The elevation of the alidade, as determined by this sight, is equal to the elevation of the point on the ground toward which the sight is taken, plus the height above that ground point of the point on the signal to which the middle cross-hair was brought, minus the difference in elevation corresponding to the horizontal distance and the vertical angle, minus the correction for curvature and refraction. Thus, the elevation of the alidade is

$$876.3+16.6-130.5-0.8=761.6 \text{ feet}$$

The various values for the sights to B and C are determined similarly. The mean of the three computed elevations of the alidade is $\dfrac{761.6+762.2+760.8}{3}=761.5$ feet, and the corresponding elevation of Sta. P is $761.5-4.1=757.4$ feet.

FIELD METHODS FOR LOCATING TOPOGRAPHY

77. Method of Radiation.—A small area can be mapped from a single setting of the plane table, the various points being located by radiation. In Fig. 17, the plane table is set up at a point designated as O and is oriented so that the plotted positions of the points A, B, C, D, and E will fall on the plane-table sheet. A point o is selected on the sheet to correspond to the station O on the ground, and the points a, b, c, d, and e are located by the method of Art. 65. With the alidade pivoted on the plotted position of the plane-table station O, a sight is taken to A and a ray is drawn along the edge of the ruler. Also, the distance from O to A is measured, this distance is laid off to scale along the ray, and the point a is plotted at the end of the measurement. Each of the points b, c, d, and e is plotted in a similar manner. On more extended surveys, this

method is used for locating topographic details from each station after the plane table has been oriented.

78. Traversing With Plane Table.—When two or more plane-table stations are to be located from one another by means of the plane table, a traverse that is somewhat similar to a transit-stadia traverse may be run. The traverse method of locating plane-table stations is shown in Fig. 24. The table

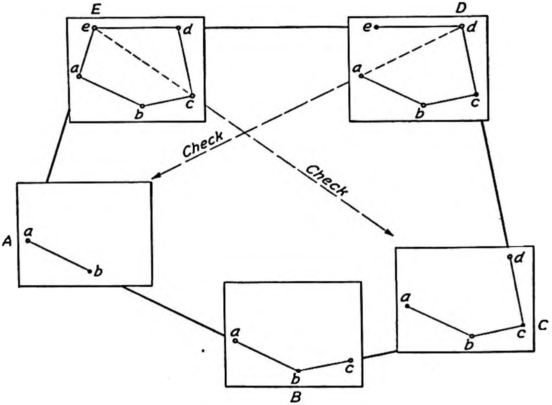


FIG. 24

is first set up at Sta. *A* and is so oriented that the map will fall on the sheet. The alidade is then pivoted at *a* on the map, the line of sight is directed toward the new station *B*, and a ray is drawn along the edge of the ruler. The distance to *B* is measured by taping or by stadia and *b* is located on the map by plotting this distance to scale. After sights have been taken to all details that are to be located from Sta. *A*, the plane table is moved to the new station *B*.

The board is oriented at Sta. *B* by placing the edge of the alidade along the line *b a* on the map and turning the table until the line of sight strikes Sta. *A* on the ground. With the edge of the alidade pivoted on *b*, the plotted position *c* of the

next station is located by directing the line of sight toward the station C on the ground, drawing the ray along the edge of the ruler, and laying off to scale the measured length BC.

This process is repeated as the board is moved to each succeeding station. The work can be checked as it progresses by backsighting to more than one point. Thus, if the sight is clear, the orientation of the board and the locations of the stations on the map can be checked at Sta. D by sighting to Sta. A. Similarly, after the board has been set up at Sta. E and oriented by backsighting to Sta. D, a sight to Sta. C will serve as a check on the orientation of the board and the locations of stations E and C. Any previously located point, such as the edge of a building or a church steeple, can be used when no station is visible. The traverse, as a whole, is checked by sighting from Sta. E to Sta. A.

For very accurate work, the allowable error of closure on the map is usually about $\frac{1}{16}$ inch. A larger error is permissible for less important work. In any case, the error of closure should be distributed among the courses of the traverse by moving the plotted position of each plane-table station a distance that is proportional to the total error.

A detail of topography can be located either by its direction and distance from a single plane-table station, as described in Art. 65, or by the method of intersection from two stations, as described in Art. 66.

79. Practical Features of Plane-Table Work.—The field methods that are used on a plane-table survey depend on the scale to which the map is being drawn and on the precision required. For example, if a survey is being made for an architect, the scale may be between 1 inch = 8 feet and 1 inch = 50 feet. Errors of 1 or 2 feet will be very apparent on maps drawn to such comparatively large scales, and consequently the board must be carefully set up so that the point on the map corresponding to the station will be directly over the station on the ground. This can be done by using the special plumbing device furnished with some plane-table outfits, or by dropping a small stone from the under side of the board at a point esti-

mated to be directly underneath the plotted point. If more than one plane-table station is required, the stations should be located by a transit-and-tape traverse. The distances to all important points, such as property corners and existing buildings, should be measured with the tape. Whenever possible, the alidade should be used as a level in determining elevations. In case the differences of elevation are fairly large, the use of a wye or dumpy level for determining elevations is preferable, because the level can be moved more rapidly than the plane table.

For intermediate-scale maps, less care is necessary in placing the plane table over the station. A few triangulation or transit-traverse stations should be plotted on the sheet before the plane-table work is started, and additional plane-table stations may be located, as needed, by traversing, by intersection, or by the three-point method. The elevations of these stations are determined by vertical angles and either stadia readings or scaled horizontal distances. Detail is located by stadia measurements or by intersection.

When a large area is to be mapped to a very small scale, it is practically imperative to run a preliminary control survey. A triangulation survey is usually advantageous for hilly terrain, and a transit-tape traverse is best for flat terrain. As longer distances are encountered, a higher percentage of the points will be located by intersection than when large or intermediate scales are used. To avoid confusion and unnecessary marking of the sheet, only the part of each ray that is in the general vicinity of the object should be drawn. However, this part should be long enough to indicate clearly the station from which the pointing was made.

The traverse plane table is used for making reconnaissance sketches for military purposes, for running traverses for small-scale maps, and for locating topographic detail that is to be transferred to a larger plane-table map. For sketching purposes, the board can be oriented by means of the trough compass with which the board is equipped. Locations are made by intersection, or by pacing or estimating distances in the proper directions.

80. Plane-Table Field Party.—A plane-table party consists of an instrumentman, generally called a topographer, and one or more rodmen. Additional helpers, such as a recorder-computer when the survey is a large one or axmen when the country is wooded, may be added as needed. The topographer makes the observations and does the plotting and sketching. The recorder-computer calculates the elevations and corrected horizontal distances; he also records any vertical angles or stadia distances that may be desirable for future reference. The rodmen should be able to judge what points must be located to represent properly the ground covered by the survey.

81. Advantages and Disadvantages of Plane-Table Method. The important advantage of the plane table, as compared with transit methods of making a topographic map, is that the map is made directly in the field and should therefore be a truer representation of the ground surface than is a map made in the office from field notes. By a graphical solution of the two-point or three-point problem, the selection of stations from which topography is to be taken is made more flexible than when the transit is used. Inaccessible points can be located, and their positions can be checked, before the party leaves the field. As the map is completed in the field, important omissions are at once obvious. By means of check lines, errors in measurements or in plotting can be detected. As field notes for directions are unnecessary, many mistakes in recording are avoided.

The principal disadvantages are that the board is cumbersome and awkward to carry. The work may be delayed by wind or rain, when work with a transit would be possible. As angles are measured graphically and field notes for distances are not always kept, numerical values of angles, and sometimes also of distances, can be obtained only by scaling them from the map. Areas must be obtained either by planimetering or by using scaled dimensions in making the necessary calculations.

ACCURACY OF PLANE-TABLE WORK

82. Errors in Plane-Table Surveys.—Inaccuracies in plane-table surveys are caused by instrumental errors, by errors in drafting, and by the instability of the board.

The instrumental errors are practically the same as those for transit-stadia work. By adjusting and properly manipulating the alidade, error from this source can be kept to a minimum. Except for large-scale maps, the error caused by not placing the station on the sheet directly over the station on the ground will cause no perceptible error.

In the case of a small-scale map, the width of a pencil line may represent many feet on the ground, and considerable care must be exercised in drawing the rays on such a map. An extremely hard, well-pointed pencil should be used for this purpose. A needle should be used in plotting station points.

Weather conditions may greatly affect the precision of a plane-table map. Stretching and shrinking of the sheet will be considerable when the weather is changeable. Error from this source can be practically eliminated by using paper sheets that are mounted on muslin or on thin sheets of aluminum. If mounted sheets are not used, all triangulation control that is to appear on a sheet should be plotted at one time.

On windy days the measurement of stadia distances may be almost impossible. If a sheltered station can be occupied, sights from the more exposed one should be left for calmer weather. Vertical angles are affected when the board is not perfectly horizontal. As many boards soon become warped, the index correction should be determined for all important pointings. The topographer must be careful not to lean on the board while sighting or plotting. To guard against possible movement of the board, the orientation should be checked at frequent intervals, particularly when new stations are being located.

83. Adjustments of Alidade.—The adjustments of the plane-table alidade are quite similar to corresponding adjustments of the transit and the wye level. As small errors in the adjust-

ment of the alidade will have no appreciable effect on the accuracy of the plane-table map, the alidade need not be adjusted with the same refinement that is required for the transit or level. The methods of making the adjustments depend somewhat on the type of alidade.

84. Adjustment of Levels on Straightedge.—For indicating when the board is level, the straightedge of the alidade may be equipped with either a circular level or two levels similar to those on the plate of a transit. The procedure in testing and adjusting the circular level or the other bubbles is as follows: The alidade is placed on the board over the tripod and the position of the straightedge is marked on the paper. The bubble or bubbles are then centered by means of the leveling attachment with which the table is provided. Now, the alidade is lifted from the board, is turned end for end, and is replaced so that the ruler occupies exactly the same space on the paper as in the initial position. If the bubbles are found to be centered in the new position, no adjustment is necessary. Otherwise, the bubbles are brought half-way back to the centers by means of adjusting screws in the connection to the straightedge. It is important that the alidade should occupy exactly the same place on the board for both directions of the telescope, because the board may be warped. The board is not moved during this adjustment.

85. Adjustment of Vertical Cross-Hair.—The vertical cross-hair of the alidade should be truly vertical when the board is leveled. The verticality of the cross-hair is tested by bringing one end of the hair on some well-defined point and noting whether or not the hair remains on that point as the telescope is raised or depressed by rotating the telescope on its transverse axis. If the hair remains on the point, no adjustment is necessary. If the hair moves off the point, two adjacent screws that hold the cross-hair ring in place in the telescope are loosened, the ring is rotated as much as seems necessary, the screws are tightened, and the test is repeated.

86. Adjustment of Fixed Telescope Level.—The adjustment of a level tube that is fixed to the telescope of the alidade

may be tested and corrected, if necessary, by the peg method. The procedure is exactly the same as that for the adjustment of the level on the telescope of a transit.

87. Adjustment of Striding Level.—If the alidade is equipped with a striding level, the procedure for testing its adjustment is as follows: The striding level is attached to the telescope, and the bubble is brought to the center of the tube by means of the tangent screw for the vertical motion of the telescope. The level is then removed from the telescope, is turned end for end, and is replaced. If the bubble again comes to the center, no adjustment is required. In case the bubble is not centered, it is brought half-way back to the center by means of the adjusting screw.

88. Adjustment of Line of Sight.—When the telescope can be rotated in its bearings, the position of the intersection of the vertical and middle horizontal cross-hairs may be tested and adjusted in the following manner: The intersection of these two hairs is brought on some well-defined point, and the telescope is rotated through an angle of 180° in its bearings. If the intersection of the hairs remains on the point, no adjustment is necessary. On the other hand, if the intersection leaves the point, each hair is brought half-way back to the point by shifting the cross-hair ring in the telescope.

89. Adjustment of Vernier on Vertical Arc.—The adjustment of the vernier for the vertical arc of an alidade telescope is tested and corrected, if necessary, as follows: The bubble in either the fixed telescope level or the striding level is centered, and the vernier reading is observed. If the reading is zero, no adjustment is necessary. Otherwise, the zero mark of the vernier is made to coincide with the zero mark of the vertical arc by loosening the screws that hold the vernier in place and tapping the vernier lightly.

90. Adjustment of Vernier Level.—If the vernier arm is equipped with a separate level, the adjustment of that level

is tested and corrected, if necessary, in the following manner: The line of sight is leveled by means of the striding level or by applying the peg method, and the vernier is set to read zero. If the bubble does not lie in the center of its tube, it is centered by means of the adjusting screw.

STADIA REDUCTION TABLE

Minutes	0°		1°		2°		3°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	100.00	.00	99.97	1.74	99.88	3.49	99.73	5.23
2	100.00	.06	99.97	1.80	99.87	3.55	99.72	5.28
4	100.00	.12	99.97	1.86	99.87	3.60	99.71	5.34
6	100.00	.17	99.96	1.92	99.87	3.66	99.71	5.40
8	100.00	.23	99.96	1.98	99.86	3.72	99.70	5.46
10	100.00	.29	99.96	2.04	99.86	3.78	99.69	5.52
12	100.00	.35	99.96	2.09	99.85	3.84	99.69	5.57
14	100.00	.41	99.95	2.15	99.85	3.89	99.68	5.63
16	100.00	.47	99.95	2.21	99.84	3.95	99.68	5.69
18	100.00	.52	99.95	2.27	99.84	4.01	99.67	5.75
20	100.00	.58	99.95	2.33	99.83	4.07	99.66	5.80
22	100.00	.64	99.94	2.38	99.83	4.13	99.66	5.86
24	100.00	.70	99.94	2.44	99.82	4.18	99.65	5.92
26	99.99	.76	99.94	2.50	99.82	4.24	99.64	5.98
28	99.99	.81	99.93	2.56	99.81	4.30	99.63	6.04
30	99.99	.87	99.93	2.62	99.81	4.36	99.63	6.09
32	99.99	.93	99.93	2.67	99.80	4.42	99.62	6.15
34	99.99	.99	99.93	2.73	99.80	4.47	99.61	6.21
36	99.99	1.05	99.92	2.79	99.79	4.53	99.61	6.27
38	99.99	1.11	99.92	2.85	99.79	4.59	99.60	6.32
40	99.99	1.16	99.92	2.91	99.78	4.65	99.59	6.38
42	99.99	1.22	99.91	2.97	99.78	4.71	99.58	6.44
44	99.98	1.28	99.91	3.02	99.77	4.76	99.58	6.50
46	99.98	1.34	99.90	3.08	99.77	4.82	99.57	6.56
48	99.98	1.40	99.90	3.14	99.76	4.88	99.56	6.61
50	99.98	1.45	99.90	3.20	99.76	4.94	99.55	6.67
52	99.98	1.51	99.89	3.26	99.75	4.99	99.55	6.73
54	99.98	1.57	99.89	3.31	99.74	5.05	99.54	6.79
56	99.97	1.63	99.89	3.37	99.74	5.11	99.53	6.84
58	99.97	1.69	99.88	3.43	99.73	5.17	99.52	6.90
60	99.97	1.74	99.88	3.49	99.73	5.23	99.51	6.96
$C =$ .75	.75	.01	.75	.02	.75	.03	.75	.05
$C =$ 1.00	1.00	.01	1.00	.03	1.00	.04	1.00	.06
$C =$ 1.25	1.25	.02	1.25	.03	1.25	.05	1.25	.08

STADIA REDUCTION TABLE—*Continued*

Minutes	4°		5°		6°		7°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	99.51	6.96	99.24	8.68	98.91	10.40	98.51	12.10
2	99.51	7.02	99.23	8.74	98.90	10.45	98.50	12.15
4	99.50	7.07	99.22	8.80	98.88	10.51	98.49	12.21
6	99.49	7.13	99.21	8.85	98.87	10.57	98.47	12.27
8	99.48	7.19	99.20	8.91	98.86	10.62	98.46	12.32
10	99.47	7.25	99.19	8.97	98.85	10.68	98.44	12.38
12	99.46	7.30	99.18	9.03	98.83	10.74	98.43	12.43
14	99.46	7.36	99.17	9.08	98.82	10.79	98.41	12.49
16	99.45	7.42	99.16	9.14	98.81	10.85	98.40	12.55
18	99.44	7.48	99.15	9.20	98.80	10.91	98.39	12.60
20	99.43	7.53	99.14	9.25	98.78	10.96	98.37	12.66
22	99.42	7.59	99.13	9.31	98.77	11.02	98.36	12.72
24	99.41	7.65	99.11	9.37	98.76	11.08	98.34	12.77
26	99.40	7.71	99.10	9.43	98.74	11.13	98.33	12.83
28	99.39	7.76	99.09	9.48	98.73	11.19	98.31	12.88
30	99.38	7.82	99.08	9.54	98.72	11.25	98.30	12.94
32	99.38	7.88	99.07	9.60	98.71	11.30	98.28	13.00
34	99.37	7.94	99.06	9.65	98.69	11.36	98.27	13.05
36	99.36	7.99	99.05	9.71	98.68	11.42	98.25	13.11
38	99.35	8.05	99.04	9.77	98.67	11.47	98.24	13.17
40	99.34	8.11	99.03	9.83	98.65	11.53	98.22	13.22
42	99.33	8.17	99.01	9.88	98.64	11.59	98.20	13.28
44	99.32	8.22	99.00	9.94	98.63	11.64	98.19	13.33
46	99.31	8.28	98.99	10.00	98.61	11.70	98.17	13.39
48	99.30	8.34	98.98	10.05	98.60	11.76	98.16	13.45
50	99.29	8.40	98.97	10.11	98.58	11.81	98.14	13.50
52	99.28	8.45	98.96	10.17	98.57	11.87	98.13	13.56
54	99.27	8.51	98.94	10.22	98.56	11.93	98.11	13.61
56	99.26	8.57	98.93	10.28	98.54	11.98	98.10	13.67
58	99.25	8.63	98.92	10.34	98.53	12.04	98.08	13.73
60	99.24	8.68	98.91	10.40	98.51	12.10	98.06	13.78
C = .75	.75	.06	.75	.07	.75	.08	.74	.10
C = 1.00	1.00	.08	1.00	.10	.99	.11	.99	.13
C = 1.25	1.25	.10	1.24	.12	1.24	.14	1.24	.16

STADIA REDUCTION TABLE—*Continued*

Minutes	8°		9°		10°		11°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	98.06	13.78	97.55	15.45	96.98	17.10	96.36	18.73
2	98.05	13.84	97.53	15.51	96.96	17.16	96.34	18.78
4	98.03	13.89	97.52	15.56	96.94	17.21	96.32	18.84
6	98.01	13.95	97.50	15.62	96.92	17.26	96.29	18.89
8	98.00	14.01	97.48	15.67	96.90	17.32	96.27	18.95
10	97.98	14.06	97.46	15.73	96.88	17.37	96.25	19.00
12	97.97	14.12	97.44	15.78	96.86	17.43	96.23	19.05
14	97.95	14.17	97.43	15.84	96.84	17.48	96.21	19.11
16	97.93	14.23	97.41	15.89	96.82	17.54	96.18	19.16
18	97.92	14.28	97.39	15.95	96.80	17.59	96.16	19.21
20	97.90	14.34	97.37	16.00	96.78	17.65	96.14	19.27
22	97.88	14.40	97.35	16.06	96.76	17.70	96.12	19.32
24	97.87	14.45	97.33	16.11	96.74	17.76	96.09	19.38
26	97.85	14.51	97.31	16.17	96.72	17.81	96.07	19.43
28	97.83	14.56	97.29	16.22	96.70	17.86	96.05	19.48
30	97.82	14.62	97.28	16.28	96.68	17.92	96.03	19.54
32	97.80	14.67	97.26	16.33	96.66	17.97	96.00	19.59
34	97.78	14.73	97.24	16.39	96.64	18.03	95.98	19.64
36	97.76	14.79	97.22	16.44	96.62	18.08	95.96	19.70
38	97.75	14.84	97.20	16.50	96.60	18.14	95.93	19.75
40	97.73	14.90	97.18	16.55	96.57	18.19	95.91	19.80
42	97.71	14.95	97.16	16.61	96.55	18.24	95.89	19.86
44	97.69	15.01	97.14	16.66	96.53	18.30	95.86	19.91
46	97.68	15.06	97.12	16.72	96.51	18.35	95.84	19.96
48	97.66	15.12	97.10	16.77	96.49	18.41	95.82	20.02
50	97.64	15.17	97.08	16.83	96.47	18.46	95.79	20.07
52	97.62	15.23	97.06	16.88	96.45	18.51	95.77	20.12
54	97.61	15.28	97.04	16.94	96.42	18.57	95.75	20.18
56	97.59	15.34	97.02	16.99	96.40	18.62	95.72	20.23
58	97.57	15.40	97.00	17.05	96.38	18.68	95.70	20.28
60	97.55	15.45	96.98	17.10	96.36	18.73	95.68	20.34
C = .75	.74	.11	.74	.12	.74	.14	.73	.15
C = 1.00	.99	.15	.99	.17	.98	.18	.98	.20
C = 1.25	1.24	.18	1.23	.21	1.23	.23	1.22	.25

STADIA REDUCTION TABLE—*Continued*

Minutes	12°		13°		14°		15°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	95.68	20.34	94.94	21.92	94.15	23.47	93.30	25.00
2	95.65	20.39	94.91	21.97	94.12	23.52	93.27	25.05
4	95.63	20.44	94.89	22.02	94.09	23.58	93.24	25.10
6	95.61	20.50	94.86	22.08	94.07	23.63	93.21	25.15
8	95.58	20.55	94.84	22.13	94.04	23.68	93.18	25.20
10	95.56	20.60	94.81	22.18	94.01	23.73	93.16	25.25
12	95.53	20.66	94.79	22.23	93.98	23.78	93.13	25.30
14	95.51	20.71	94.76	22.28	93.95	23.83	93.10	25.35
16	95.49	20.76	94.73	22.34	93.93	23.88	93.07	25.40
18	95.46	20.81	94.71	22.39	93.90	23.93	93.04	25.45
20	95.44	20.87	94.68	22.44	93.87	23.99	93.01	25.50
22	95.41	20.92	94.66	22.49	93.84	24.04	92.98	25.55
24	95.39	20.97	94.63	22.54	93.82	24.09	92.95	25.60
26	95.36	21.03	94.60	22.60	93.79	24.14	92.92	25.65
28	95.34	21.08	94.58	22.65	93.76	24.19	92.89	25.70
30	95.32	21.13	94.55	22.70	93.73	24.24	92.86	25.75
32	95.29	21.18	94.52	22.75	93.70	24.29	92.83	25.80
34	95.27	21.24	94.50	22.80	93.67	24.34	92.80	25.85
36	95.24	21.29	94.47	22.85	93.65	24.39	92.77	25.90
38	95.22	21.34	94.44	22.91	93.62	24.44	92.74	25.95
40	95.19	21.39	94.42	22.96	93.59	24.49	92.71	26.00
42	95.17	21.45	94.39	23.01	93.56	24.55	92.68	26.05
44	95.14	21.50	94.36	23.06	93.53	24.60	92.65	26.10
46	95.12	21.55	94.34	23.11	93.50	24.65	92.62	26.15
48	95.09	21.60	94.31	23.16	93.47	24.70	92.59	26.20
50	95.07	21.66	94.28	23.22	93.45	24.75	92.56	26.25
52	95.04	21.71	94.26	23.27	93.42	24.80	92.53	26.30
54	95.02	21.76	94.23	23.32	93.39	24.85	92.49	26.35
56	94.99	21.81	94.20	23.37	93.36	24.90	92.46	26.40
58	94.97	21.87	94.17	23.42	93.33	24.95	92.43	26.45
60	94.94	21.92	94.15	23.47	93.30	25.00	92.40	26.50
C = .75	.73	.16	.73	.18	.73	.19	.72	.20
C = 1.00	.98	.22	.97	.23	.97	.25	.96	.27
C = 1.25	1.22	.27	1.22	.29	1.21	.31	1.20	.33

STADIA REDUCTION TABLE—*Continued*

Minutes	16°		17°		18°		19°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	92.40	26.50	91.45	27.96	90.45	29.39	89.40	30.78
2	92.37	26.55	91.42	28.01	90.42	29.44	89.36	30.83
4	92.34	26.59	91.39	28.06	90.38	29.48	89.33	30.87
6	92.31	26.64	91.35	28.10	90.35	29.53	89.29	30.92
8	92.28	26.69	91.32	28.15	90.31	29.58	89.26	30.97
10	92.25	26.74	91.29	28.20	90.28	29.62	89.22	31.01
12	92.22	26.79	91.26	28.25	90.24	29.67	89.18	31.06
14	92.19	26.84	91.22	28.30	90.21	29.72	89.15	31.10
16	92.15	26.89	91.19	28.34	90.18	29.76	89.11	31.15
18	92.12	26.94	91.16	28.39	90.14	29.81	89.08	31.19
20	92.09	26.99	91.12	28.44	90.11	29.86	89.04	31.24
22	92.06	27.04	91.09	28.49	90.07	29.90	89.00	31.28
24	92.03	27.09	91.06	28.54	90.04	29.95	88.97	31.33
26	92.00	27.13	91.02	28.58	90.00	30.00	88.93	31.38
28	91.97	27.18	90.99	28.63	89.97	30.04	88.89	31.42
30	91.93	27.23	90.96	28.68	89.93	30.09	88.86	31.47
32	91.90	27.28	90.92	28.73	89.90	30.14	88.82	31.51
34	91.87	27.33	90.89	28.77	89.86	30.18	88.78	31.56
36	91.84	27.38	90.86	28.82	89.83	30.23	88.75	31.60
38	91.81	27.43	90.82	28.87	89.79	30.28	88.71	31.65
40	91.77	27.48	90.79	28.92	89.76	30.32	88.67	31.69
42	91.74	27.52	90.76	28.96	89.72	30.37	88.64	31.74
44	91.71	27.57	90.72	29.01	89.69	30.41	88.60	31.78
46	91.68	27.62	90.69	29.06	89.65	30.46	88.56	31.83
48	91.65	27.67	90.66	29.11	89.61	30.51	88.53	31.87
50	91.61	27.72	90.62	29.15	89.58	30.55	88.49	31.92
52	91.58	27.77	90.59	29.20	89.54	30.60	88.45	31.96
54	91.55	27.81	90.55	29.25	89.51	30.65	88.41	32.01
56	91.52	27.86	90.52	29.30	89.47	30.69	88.38	32.05
58	91.48	27.91	90.49	29.34	89.44	30.74	88.34	32.09
60	91.45	27.96	90.45	29.39	89.40	30.78	88.30	32.14
C = .75	.72	.21	.72	.23	.71	.24	.71	.25
C = 1.00	.96	.28	.95	.30	.95	.32	.94	.33
C = 1.25	1.20	.36	1.19	.38	1.19	.40	1.18	.42

STADIA REDUCTION TABLE—*Continued*

Minutes	20° Hor. Dist.	20° Diff. Elev.	21° Hor. Dist.	21° Diff. Elev.	22° Hor. Dist.	22° Diff. Elev.	23° Hor. Dist.	23° Diff. Elev.
0	88.30	32.14	87.16	33.46	85.97	34.73	84.73	35.97
2	88.26	32.18	87.12	33.50	85.93	34.77	84.69	36.01
4	88.23	32.23	87.08	33.54	85.89	34.82	84.65	36.05
6	88.19	32.27	87.04	33.59	85.85	34.86	84.61	36.09
8	88.15	32.32	87.00	33.63	85.80	34.90	84.57	36.13
10	88.11	32.36	86.96	33.67	85.76	34.94	84.52	36.17
12	88.08	32.41	86.92	33.72	85.72	34.98	84.48	36.21
14	88.04	32.45	86.88	33.76	85.68	35.02	84.44	36.25
16	88.00	32.49	86.84	33.80	85.64	35.07	84.40	36.29
18	87.96	32.54	86.80	33.84	85.60	35.11	84.35	36.33
20	87.93	32.58	86.77	33.89	85.56	35.15	84.31	36.37
22	87.89	32.63	86.73	33.93	85.52	35.19	84.27	36.41
24	87.85	32.67	86.69	33.97	85.48	35.23	84.23	36.45
26	87.81	32.72	86.65	34.01	85.44	35.27	84.18	36.49
28	87.77	32.76	86.61	34.06	85.40	35.31	84.14	36.53
30	87.74	32.80	86.57	34.10	85.36	35.36	84.10	36.57
32	87.70	32.85	86.53	34.14	85.31	35.40	84.06	36.61
34	87.66	32.89	86.49	34.18	85.27	35.44	84.01	36.65
36	87.62	32.93	86.45	34.23	85.23	35.48	83.97	36.69
38	87.58	32.98	86.41	34.27	85.19	35.52	83.93	36.73
40	87.54	33.02	86.37	34.31	85.15	35.56	83.89	36.77
42	87.51	33.07	86.33	34.35	85.11	35.60	83.84	36.80
44	87.47	33.11	86.29	34.40	85.07	35.64	83.80	36.84
46	87.43	33.15	86.25	34.44	85.02	35.68	83.76	36.88
48	87.39	33.20	86.21	34.48	84.98	35.72	83.72	36.92
50	87.35	33.24	86.17	34.52	84.94	35.76	83.67	36.96
52	87.31	33.28	86.13	34.57	84.90	35.80	83.63	37.00
54	87.27	33.33	86.09	34.61	84.86	35.85	83.59	37.04
56	87.24	33.37	86.05	34.65	84.82	35.89	83.54	37.08
58	87.20	33.41	86.01	34.69	84.77	35.93	83.50	37.12
60	87.16	33.46	85.97	34.73	84.73	35.97	83.46	37.16
C = .75	.70	.26	.70	.27	.69	.29	.69	.30
C = 1.00	.94	.35	.93	.37	.92	.38	.92	.40
C = 1.25	1.17	.44	1.16	.46	1.15	.48	1.15	.50

STADIA REDUCTION TABLE—*Continued*

Minutes	24°		25°		26°		27°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	83.46	37.16	82.14	38.30	80.78	39.40	79.39	40.45
2	83.41	37.20	82.09	38.34	80.74	39.44	79.34	40.49
4	83.37	37.23	82.05	38.38	80.69	39.47	79.30	40.52
6	83.33	37.27	82.01	38.41	80.65	39.51	79.25	40.55
8	83.28	37.31	81.96	38.45	80.60	39.54	79.20	40.59
10	83.24	37.35	81.92	38.49	80.55	39.58	79.15	40.62
12	83.20	37.39	81.87	38.53	80.51	39.61	79.11	40.66
14	83.15	37.43	81.83	38.56	80.46	39.65	79.06	40.69
16	83.11	37.47	81.78	38.60	80.41	39.69	79.01	40.72
18	83.07	37.51	81.74	38.64	80.37	39.72	78.96	40.76
20	83.02	37.54	81.69	38.67	80.32	39.76	78.92	40.79
22	82.98	37.58	81.65	38.71	80.28	39.79	78.87	40.82
24	82.93	37.62	81.60	38.75	80.23	39.83	78.82	40.86
26	82.89	37.66	81.56	38.78	80.18	39.86	78.77	40.89
28	82.85	37.70	81.51	38.82	80.14	39.90	78.73	40.92
30	82.80	37.74	81.47	38.86	80.09	39.93	78.68	40.96
32	82.76	37.77	81.42	38.89	80.04	39.97	78.63	40.99
34	82.72	37.81	81.38	38.93	80.00	40.00	78.58	41.02
36	82.67	37.85	81.33	38.97	79.95	40.04	78.54	41.06
38	82.63	37.89	81.28	39.00	79.90	40.07	78.49	41.09
40	82.58	37.93	81.24	39.04	79.86	40.11	78.44	41.12
42	82.54	37.96	81.19	39.08	79.81	40.14	78.39	41.16
44	82.49	38.00	81.15	39.11	79.76	40.18	78.34	41.19
46	82.45	38.04	81.10	39.15	79.72	40.21	78.30	41.22
48	82.41	38.08	81.06	39.18	79.67	40.24	78.25	41.26
50	82.36	38.11	81.01	39.22	79.62	40.28	78.20	41.29
52	82.32	38.15	80.97	39.26	79.58	40.31	78.15	41.32
54	82.27	38.19	80.92	39.29	79.53	40.35	78.10	41.35
56	82.23	38.23	80.87	39.33	79.48	40.38	78.06	41.39
58	82.18	38.26	80.83	39.36	79.44	40.42	78.01	41.42
60	82.14	38.30	80.78	39.40	79.39	40.45	77.96	41.45
C = .75	.68	.31	.68	.32	.67	.33	.67	.35
C = 1.00	.91	.41	.90	.43	.89	.45	.89	.46
C = 1.25	1.14	.52	1.13	.54	1.12	.56	1.11	.58

STADIA REDUCTION TABLE—*Continued*

Minutes	28°		29°		30°	
	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.	Hor. Dist.	Diff. Elev.
0	77.96	41.45	76.50	42.40	75.00	43.30
2	77.91	41.48	76.45	42.43	74.95	43.33
4	77.86	41.52	76.40	42.46	74.90	43.36
6	77.81	41.55	76 35	42.49	74.85	43.39
8	77.77	41.58	76.30	42.53	74.80	43.42
10	77.72	41.61	76.25	42.56	74.75	43.45
12	77.67	41.65	76.20	42.59	74.70	43.47
14	77.62	41.68	76.15	42.62	74.65	43.50
16	77.57	41.71	76.10	42.65	74.60	43.53
18	77.52	41.74	76.05	42.68	74.55	43.56
20	77.48	41.77	76.00	42.71	74.49	43.59
22	77.42	41.81	75.95	42.74	74.44	43.62
24	77.38	41.84	75.90	42.77	74.39	43.65
26	77.33	41.87	75.85	42.80	74.34	43.67
28	77.28	41.90	75.80	42.83	74.29	43.70
30	77.23	41.93	75.75	42.86	74.24	43.73
32	77.18	41.97	75.70	42.89	74.19	43.76
34	77.13	42.00	75.65	42.92	74.14	43.79
36	77.09	42.03	75.60	42.95	74.09	43.82
38	77.04	42.06	75.55	42.98	74.04	43.84
40	76.99	42.09	75.50	43.01	73.99	43.87
42	76.94	42.12	75.45	43.04	73.93	43.90
44	76.89	42.15	75.40	43.07	73.88	43.93
46	76.84	42.19	75.35	43.10	73.83	43.95
48	76.79	42.22	75.30	43.13	73.78	43.98
50	76.74	42.25	75.25	43.16	73.73	44.01
52	76.69	42.28	75.20	43.18	73.68	44.04
54	76.64	42.31	75.15	43.21	73.63	44.07
56	76.59	42.34	75.10	43.24	73.58	44.09
58	76.55	42.37	75.05	43.27	73.52	44.12
60	76.50	42.40	75.00	43.30	73.47	44.15
C = .75	.66	.36	.65	.37	.65	.38
C = 1.00	.88	.48	.87	.49	.86	.51
C = 1.25	1.10	.60	1.09	.62	1.08	.63

TOPOGRAPHIC SURVEYING

Serial 2905-2 Edition 1

INTRODUCTION

GENERAL METHODS

TOPOGRAPHIC MAPS

1. **Use of Topographic Maps.**—In the usual land surveys it is necessary to find only the directions and lengths of lines between various selected points. In topographic surveying, however, it is required to determine not only the relative positions of points on the surface of the earth, but also their elevations above an arbitrarily assumed level surface called the *datum*. The representation of the features of a portion of the earth's surface on a plane surface, showing the horizontal distances and directions between points and their elevations, is a topographic map. Furthermore, the representation of the differences of elevations is called the *relief*. Although the primary purpose of a topographic map is to depict the relief, it also shows both the natural and artificial features of the country represented, as rivers, lakes, vegetation, roads, railroads, towns, houses, fences, lines indicating political boundaries, and private-property lines. The features of the ground are delineated on the map by means of conventional signs.

The topographic map has many uses. It is a help in the planning and designing of most engineering projects, and is indispensable in many cases, such as the layout of industrial plants, the location of railways and highways, the design of irrigation and drainage systems, the development of water power, city planning, and landscape architecture. It is also of great importance in time of war in the direction of military operations.

Topographic surveys range from a simple survey of a small area, as a city lot, to an extensive, complicated survey covering an entire country.

2. Scale of Map and Accuracy of Measurements.—A topographic map is a representation, on a very small area, of a portion of the surface of the earth. The distance between any two points shown on the map has a definite ratio to the distance between the corresponding two points on the ground, and this ratio is known as the scale of the map. It is expressed in various ways. For example, if 1 inch on the map represents 100 feet on the ground, the scale may be stated as 1 inch = 100 feet. However, since 100 feet = 1,200 inches, the scale may also be expressed as 1:1,200, or as $\frac{1}{1,200}$.

The scale to which a map is plotted depends primarily on the purpose of the map, that is, on the desired degree of accuracy with which distances could later be measured or scaled on the map. On the other hand, the required scale of the proposed map determines largely the necessary accuracy of the field measurements that are made for the sole purpose of plotting the map. Thus, when the scale is 1 inch = 100 feet, distances can be plotted accurately to the nearest 1 foot, whereas for a scale of 1 inch = 1,000 feet the greatest accuracy in plotting that can reasonably be expected is to the nearest 10 feet. Therefore, the accuracy of the measurements in the field is often governed by the scale of the map.

3. Contours.—The relief on a map is most commonly represented by means of lines called contours, or contour lines. A contour is an imaginary line on the surface of the earth all points of which have the same elevation; in other words, it is the line of intersection of the surface of the ground with a level surface. The shore line of a quiet body of water, as a lake or a pond, is an illustration of a contour on the ground. A distinction is sometimes made by calling the imaginary lines on the ground contours, and the lines that represent them on the map contour lines. However, the lines on the map are often referred to simply as contours.

The manner in which the relative elevations of various points on a given surface can be represented by contour lines is illustrated in Fig. 1. Let *MPNO* represent the top view of a hill projected on a horizontal plane, and let the irregular lines *aa*, *bb*, *cc*, *dd*, and *ee* represent contour lines around the hill at elevations of 10, 20, 30, 40, and 50 feet, respectively. In other words, *aa* is an imaginary line around the hill through all points at elevation 10; if the hill were surrounded by water, the surface of which is at that elevation, then the line *aa* would represent the

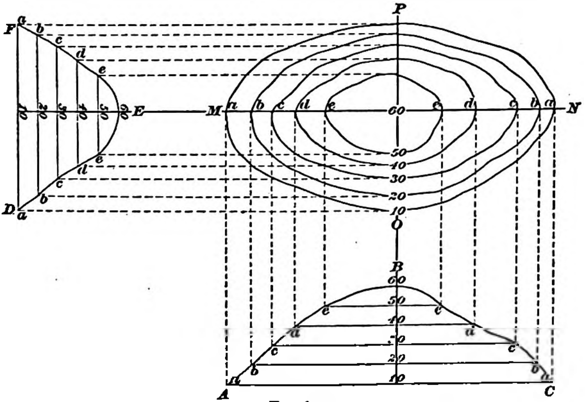


FIG. 1

shore line of the water. Similarly, if the water surface were to rise to elevation 20, the shore line would be represented by the contour *bb*.

4. Plotting of Profile from Contour Lines.—A common use of contours is in the construction of a vertical section or profile along some line on the map. For instance, in Fig. 1, the contours in the horizontal projection *MPNO* may be used to plot the vertical section *ABC* along the line *MN* or the vertical section *DEF* along *OP*.

To construct the profile *ABC*, it is first necessary to draw a series of lines parallel to *MN* and spaced at equal distances apart,

the vertical distance between each two consecutive lines representing to some convenient scale the difference in elevation between adjacent contour lines, which in this case is 10 feet. The points of intersection of contour *aa* and the line *MN* are projected onto the parallel line representing an elevation of 10 feet and are marked *a*. Similarly, the intersections of contour *bb* and *MN* are projected to the 20-foot line and the points are marked *b*. When all the intersection points have been transferred to the vertical sections they are connected by a continuous freehand line which represents the surface of the ground along the line *MN*. The profile *DEF* is drawn in the same way by using the intersections of the contours and the line *PO*.

5. **Contour Intervals.**—The difference in elevation or vertical distance between two adjacent contour lines is called the contour interval. It is usually taken as some whole number of feet, as 1 foot, 2 feet, 5 feet, or 20 feet. Thus, the contour interval in Fig. 1 is 10 feet. The choice of contour interval depends upon the accuracy desired in reading elevations from the map. The error in the elevation of any point should not be greater than half the contour interval. Hence, if it is desired to read elevations from the map to the nearest foot, the interval should not be greater than 2 feet. When the interval is 5 feet, it should be possible to read elevations with an error not greater than $2\frac{1}{2}$ feet, etc.

A test of the accuracy of the contours shown on a map can be made as follows: A line of levels is run along some line located on the ground, and a profile is plotted from the level notes. Also, the line is drawn in its proper position on the map, and a profile is plotted from the intersections with the contours, as described in the preceding article. The two profiles should check within the specified limits of accuracy.

METHODS OF CONTROL

6. **Control Points.**—To make a topographic survey, points must be established from which the details can be located. If the survey is very small, as a city lot, all points may be located from a single instrument station. The larger the survey the

more points are required. Such points must be located with respect to each other with an accuracy sufficient for the purposes of the work. This system of points carefully located by directions, horizontal distances, and elevations is the control. It is the framework or outline for the detailed surveys. The location of the points by directions and horizontal measurements is the *horizontal control.* The location of points by running lines of levels and thereby establishing the elevations is the *vertical control.*

7. Horizontal Control by Traverses.—The method to be adopted for horizontal control depends on the character of the country and the extent of the area to be covered. In flat country, or in heavily timbered areas where it is difficult to see more than a short distance from any point. horizontal control may be best obtained by means of traverses. The distances are measured on the ground with a tape and the angles at the vertexes of the traverses are measured with a transit. To provide checks in the work it is desirable to lay out the traverses in a series of connected, closed circuits. If the area is small a single circuit may be sufficient. If the character of the area is such that closed circuits are not convenient, checks for distances may be obtained by repeating measurements, and checks for direction by making astronomical observations at frequent intervals along the traverse line. When traverses are used for horizontal control, the courses are run along lines that are easily accessible and the transit stations are established in suitable locations. Ridge lines and valley lines provide natural control points from which to make the detail surveys.

8. Horizontal Control by Triangulation.—In a large area of mountainous or hilly country, horizontal control is best obtained by a system of triangulation. The whole area is covered with a network of triangles, with sides several miles in length, the points forming the vertexes of the triangles being located in prominent, intervisible places. The angles in each triangle and the azimuth and length of one of the lines forming a side are measured in the field. This side is called the *base line.*

Starting with the base line, and using the observed angles in the triangles, the lengths and directions of all the lines can be computed by the principles of trigonometry. If the points selected are too far apart to furnish a sufficient number of instrument stations for locating the details of the survey, other points can then be located by a second system of triangles of shorter sides, connected to the first system, or by running a traverse or series of traverses between triangulation points. In the first triangulation system with long sides the measurements are made with high precision. In the next system of shorter sides connected to the first, the measurements are made with less precision.

The United States Coast and Geodetic Survey classifies triangulation surveys according to the precision of the work as first-order, second-order, and third-order. First-order triangulation of high precision is used for fundamental control throughout the country and for accurate city surveys, as in New York. Second-order triangulation is used for the framework of an isolated region, for the connection of first-order and third-order triangulation, and for the detailed control over important economic areas. In 1928 it was decided that the entire Coastal main-scheme triangulation in the United States should be made of second-order accuracy. Third-order triangulation is used primarily along rivers and inlets where a connection to a higher precision triangulation can be made at least every 50 miles.

9. Vertical Control.—For vertical control, lines of levels are run over the area and bench marks are established at frequent intervals. In the United States Geological Survey, bench marks are set approximately three miles apart along these lines. If a triangulation system is used for horizontal control, bench marks are established at or near the triangulation stations. When traverses are used, the lines of levels usually follow the traverses and the traverse stations are marked with elevations for use as bench marks.

An assumed datum may be used for the survey, but if the area covered is very extensive it is desirable to use the mean sea level as the datum. All government bench marks established by precise levels are referred to sea level.

The grade of work in running levels for vertical control should compare with that of the work for horizontal control. Bench-mark elevations along lines of first-order control should be established with a higher degree of accuracy than those along lines of a lower order of control. The United States Geological Survey has three grades of work in leveling, for which the maximum permissible errors in feet are:

first order, $0.017\sqrt{\text{distance in miles}}$;
second order, $0.035\sqrt{\text{distance in miles}}$;
third order, $0.05\sqrt{\text{distance in miles}}$.

METHODS OF LOCATING DETAILS

10. General Considerations.—The choice of instruments and methods to be used in the location of details depends on the character of the ground and the purpose of the survey. The instruments most generally used are the stadia transit and the plane table; the level is sometimes used in conjunction with either instrument. Aerial photography is now being used for surveying large tracts of land. The choice between the transit and plane table for taking topography is to some extent a personal one. A topographer whose experience has been mainly with a certain instrument and a particular method, is more proficient in using that method than he would be in trying one with which he is not familiar.

11. Surveys of Small and Large Areas.—In the survey of a small tract either by plane table or by transit, a closed traverse is usually established as a framework. The magnetic bearing of any line of the traverse can be observed and the azimuth of the line can be calculated from its bearing, and this azimuth can be taken as a basis from which to determine the azimuths of the other lines of the survey. The horizontal distances and the elevations can be found by suitable methods consistent with the desired accuracy. The elevations may be referred to an assumed datum, or they may be taken from a convenient bench mark if it is desired to tie up with actual elevations. In order to establish the topographic features, the transit or plane table is set up

at each station along the traverse and sights are taken to locate the various details.

In the survey of a large area, the first step is to complete the horizontal and vertical control surveys. These furnish lines with known azimuths and bench marks with known elevations to which the detail surveys are referred. The next step is to run auxiliary stadia traverses, each starting from one of the control points and closing on another, in order to furnish the necessary instrument stations for filling in the topography over the entire area. The work of running these traverses and taking the topography from the stations established on them is similar to that described for a survey of a small area.

12. Transit-Stadia and Plane-Table Methods.—Either the transit-stadia method or the plane-table method may be used for locating topography where a high degree of accuracy is not required. Certain field conditions favor the use of the transit, and others make the plane table preferable. In areas covered with timber, brush, or any growth that obstructs the view, and where large numbers of definite points are to be located, the conditions are more favorable to the use of the transit and stadia method. In this method, points are located by establishing directions by means of the transit and by determining horizontal distances and differences in elevation by means of stadia-rod readings and vertical angles. The method is well adapted to the making of general topographic surveys of considerable extent, as for preliminary railroad location.

The plane table is adapted to use in the open country where there are many indefinite details to be located. It is useful in country where the three-point problem can be used to orient the table and locate the station occupied. In the plane-table method, points are located by establishing directions, horizontal distances, and differences in elevation by means of the alidade and stadia-rod readings. The points so located are at once plotted on the sheet attached to the table, thus completing the map in the field without the intermediate process of recording notes for angles and distances. This method is extensively used by the United States Coast and Geodetic Survey and the United States Geo-

logical Survey for filling in the details after the control survey has been made by other means. It is also adapted for smaller surveys, such as that of a park in which it is desired to locate numerous objects within a small area, and for surveys for rough maps that must be made hurriedly, only the most important points being located accurately and the other features being sketched in by eye.

13. **Transit, Tape, and Level Method.**—On small areas or on long narrow areas, where a fairly high degree of accuracy is desired, the transit, tape, and level may be used together. In this method, objects are located by obtaining the azimuths with a transit, measuring horizontal distances with a tape, and determining the elevations with a wye level.

14. **Side-Shot and Cross-Section Methods.**—There are two main methods for locating contour points and details regardless of the instruments used. One method consists of locating points in all directions around the set-up, so that the area visible from the instrument will be completely surveyed with one set-up. Sights on these various points are known as side shots and the procedure may be called the side-shot method or the random-shot method.

Another method for locating points is one that is used frequently in railroad surveys and for limited areas such as parks and cemeteries. The traverse lines are taken as lines of reference for the location of points. At suitable intervals along each line, generally at each 100-foot station, cross-sections are established at right angles to the line and on both sides of it, and points are located on these sections. This system of location is known as the cross-section method.

METHODS OF LOCATING CONTOURS

15. **Methods Used.**—The two common systems for locating contours are the *direct-location method* and the *interpolation method*. Both systems may be used with either the side-shot method or the cross-section method.

16. **Direct Location of Contours.**—In cases where the contour location is the important factor, and the ground surface con-

sists of gentle even-sloping hills and valleys, the direct-location method is preferable; that is, points on the contours are located directly on the ground. In this method a level is used for determining elevations. It is first necessary to compute the rod reading which will give the desired contour elevation. The rodman sets the target to that reading and places the rod at a point on the ground which he judges to be on the contour. Then the leveler signals him to move the rod up or down hill until the cross-wire of the level cuts the target on the rod, and the position of the rod and the contour elevation are recorded. Enough of these points at each elevation are located to permit the drawing of an accurate contour by connecting the points.

17. Contour Location by Interpolation.—When the ground surface is very irregular and when the results of the survey may be used for plotting accurate profiles, the method of interpolation is used for locating contours. In this method, the rodman picks points at which there are breaks in the slope of the ground surface. These points are located and their elevations determined, usually by stadia. Enough points are chosen to enable the topographer to fill in the contours. The ground between any two adjacent points is assumed to have a constant slope, and the distances to the contour lines from the points are, therefore, proportional to the corresponding differences in elevation. Thus, if two points have elevations of 24.2 feet and 30.9 feet, respectively, and the horizontal distance between them is 175 feet, the difference in elevation between the two points is $30.9-24.2=6.7$ feet, and the rate of slope along the line joining the points is $\frac{175}{6.7}$, or 26.1, feet horizontal for 1 foot difference in elevation. As the 25-foot contour is $25.0-24.2=.8$ foot, above the lower point, the horizontal distance from that point to the 25-foot contour is $26.1\times.8=21$ feet. Similarly, the horizontal distance from that point to the 30-foot contour is $26.1\times(30-24.2)=151$ feet.

LOCATING TOPOGRAPHIC FEATURES

SIDE-SHOT METHOD

TOPOGRAPHY BY TRANSIT AND STADIA

18. **Preliminary Procedure.**—In Fig. 2 is represented a small tract of land that is to be surveyed by the transit and stadia method. In a previous survey the magnetic bearing of the part of Mead Road that is south of Mill Road, as observed from the intersection of the center lines of the two roads, was found to be S 7°30′ W and the elevation of the intersection was determined as 173.2 feet. It is now required to locate the brook forming the eastern boundary of the tract, and the roads, fences, and buildings, and also to show in a general way the relief of the tract. The party consists of an observer, a recorder, and two rodmen. The notes for the survey are as given on pages 14, 15, 16, and 17.

The first station of the new traverse is located at a point in the center of Mead Road at the angle south of Mill Road, which point is Station 1 of the survey, as marked in the notes and in Fig. 2. It is decided to close the survey at the intersection of the center lines of Mead and Mill Roads, and, since the number of intervening stations is not yet known, this point is temporarily marked x.

19. **Details of Survey.**—If azimuths are measured from the north point, the azimuth of the center line of Mead Road from Station 1 to x is 7°30′, since the bearing of this line is N 7°30′ E. With the transit set at Station 1, the vernier is set at 7°30′, the telescope is directed to the rod held on the point x, and the lower clamp is set. By this operation, the transit is oriented on the center line of Mead Road. The stadia reading of 6.30 is taken and called out to the recorder, the vertical angle of $-0°23'$ is read and called out, and the azimuth, stadia reading, and vertical angle are recorded in the notebook as shown on page 14. At

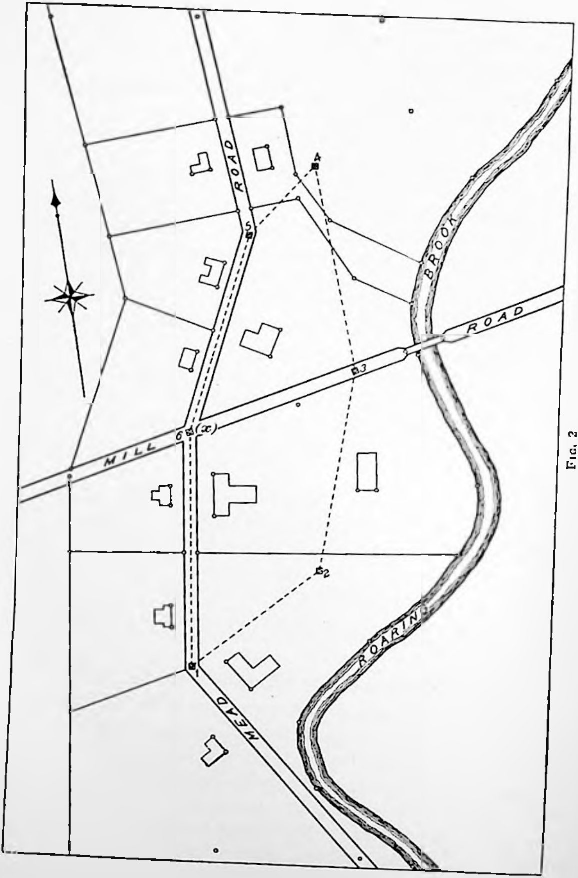


FIG. 2

the same time a scaled drawing is begun opposite the written notes, showing the relation of the various points on the ground. This sketch is filled in as the survey progresses, with each point designated by the proper letter.

After the values for point *x* have been recorded, the upper clamp is loosened and the telescope is directed to the stadia rod held successively on the points to be located. The stadia rod, vertical angle, and azimuth are read and recorded for each sight. These sights, or side shots, are preferably taken in succession by turning to the right, as continuous turning lessens the possibility of error; but this plan cannot usually be adhered to, as it is desirable that the rodmen shall cover the ground with as little unnecessary walking as possible. Points to which side shots are taken are commonly designated in the notes by the small letters of the alphabet, beginning with *a* at each instrument point. In order to locate a building, it is generally sufficient to determine the directions and lengths of the lines from a traverse station to two adjacent corners of the building and to measure the dimensions of the building.

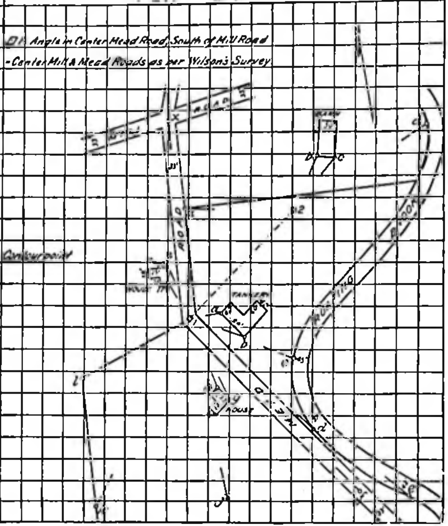
The points *a* and *b*, to which side shots are taken from Station 1, are two adjacent corners of the tannery, the dimensions of the building being measured by the rodmen or by the recorder and a rodman; points *c*, *d*, and *e* are on the edge of Roaring Brook; *f* is in the center of Mead Road; *g* and *h* are corners of the house south of the station occupied; *j* is a point where the slope of the ground changes and is marked contour point, or *c.p.*, as it is used only to locate the contours of the tract; points *k* and *l* locate the fence line, and *m* and *n* the house north of Station 1. In this type of survey contour points are located only when the slope of the ground is such that it will not be accurately represented by contours drawn from the elevations of the other points that have been established.

All the desired details adjacent to Station 1 having been located, one of the rodmen selects a position suitable for Station 2. Before sighting to this point, the observer sets the vernier again at 7°30′ and checks the position of the instrument by sighting again on the rod held at point *x*. If it is found that the instrument has moved slightly out of position, the telescope is

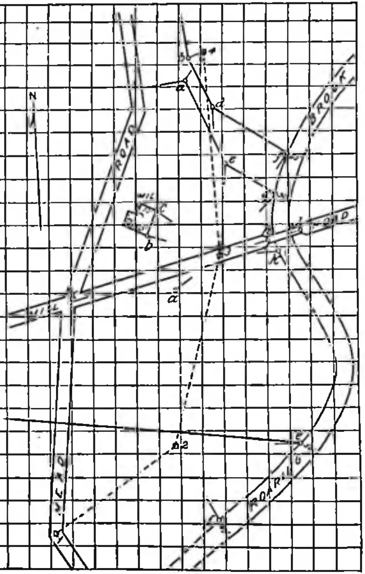
Stadia Survey of Meadville Suburb
May 27th 19__

Observer-J.R.Gavin
Recorder-Gonover
Rodmen — Griffith, Steele

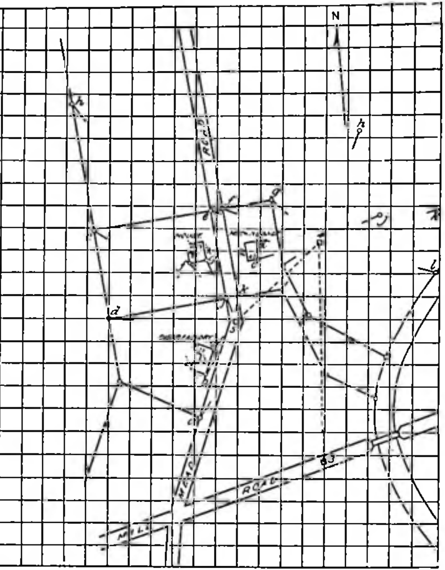
Sta	Azimuth	Stadia	Vertical Angle	Horiz.Distance	Elevation
	Readings	from ⊡1,	Elev. 177.42.		
x	7° 30′	6.30	− 0° 23′	631	173.2
a	90° 43′	.91	− 2° 04′	92	174.1
b	120° 18′	1.66	− 2° 12′	167	171.0
c	126° 31′	3.15	− 2° 06′	316	165.8
d	143° 43′	4.60	− 1° 25′	461	166.0
e	141° 32′	7.85	− 0° 38′	786	168.7
f	148° 04′	6.74	− 0° 34′	675	170.7
g	167° 19′	2.47	− 0° 50′	248	173.8
h	172° 22′	1.97	− 1° 20′	198	172.8
i	181° 17′	4.99	+ 0° 12′	500	179.2
k	221° 45′	5.79	+ 1° 02′	580	187.8
l	256° 01′	3.47	+ 1° 50′	348	188.5
m	342° 03′	1.17	+ 1° 16′	118	180.0
n	350° 16′	1.71	+ 0° 52′	172	180.0
⊡2	60° 00′	4.21	− 1° 35′	M = 422	165.77
	Readings	from ⊡2	Elev 165.77.		
⊡1		4.21	+ 1° 34′		
a	286° 01′	3.21	+ 2° 01′	322	177.1
b	32° 02′	2.36	− 0° 48′	237	162.5
c	41° 49′	2.60	− 1° 03′	261	161.0
d	68° 32′	4.61	− 1° 22′	462	154.8



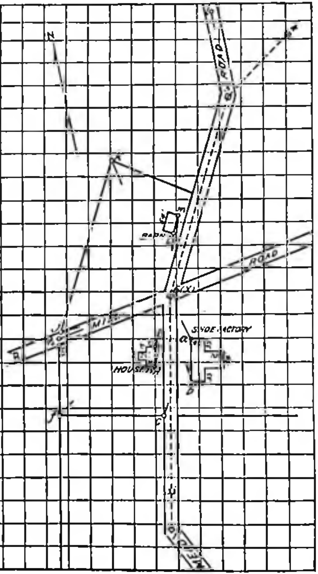
Sta.	Azimuth	Stadia	Vertical Angle	Hor Distance	Elevation	
	Readings	from □2	(cont) Elev. =	165.77.		
e	90° 45'	3.59	− 1° 18'	360	157.6	
f	154° 53'	2.28	− 0° 54'	229	162.2	
□3	17° 30'	5.45	+ 0° 17'	M = 545	168.61	
	Readings	from □3,	Elev. = 168.61.			
□2		5.44	− 0° 19'			
a	245° 30'	1.73	− 1° 53'	174	162.9	Contour Point
x	256° 45'	4.58	+ 0° 40'	459	173.9	Tie Line
b	287° 43'	2.19	− 1° 16'	220	163.7	
c	307° 24'	2.21	− 1° 41'	222	162.1	
d	359° 04'	4.06	− 1° 26'	407	158.4	
e	7° 33'	2.49	− 2° 50'	249	156.3	
f	38° 21'	3.39	− 3° 07'	339	150.1	
g	47° 20'	2.31	− 4° 18'	231	151.2	
1	76° 45'	2.39	− 0° 20'	240	167.2	
h	76° 46'	1.39	− 0° 34'	140	167.2	
k	82° 52'	1.66	− 5° 41'	165	152.2	
□4	357° 35'	5.59	− 0° 46'	M = 559	161.29	
	Readings	from □4,	Elev. = 161.29.			
□3		5.57	+ 0° 44'			
a	215° 41'	0.98		97	161.3	
b	255° 10'	0.53	+ 1° 13'	54	162.5	



Sta.	Azimuth	Stadia	Vertical Angle	Horiz. Distance	Elevation
	Readings	from ⊡4,	(cont) Elev =	161.29	
c	275° 17′	1.09	+ 1° 19′	110	163.8
d	301° 11′	1.34	+ 0° 54′	135	163.4
e	303° 46′	2.84	+ 0° 47′	285	165.2
f	308° 36′	2.57	+ 0° 43′	258	164.5
g	339° 25′	1.79	+ 0° 08′	180	161.7
h	32° 19′	4.29	− 1° 24′	430	150.8
j	70° 03′	2.67	− 1° 38′	268	153.7
k	77° 32′	6.61	− 1° 21′	662	145.7
l	102° 12′	4.03	− 1° 54′	404	147.9
⊡5	230° 54′	2.51	+ 0° 44′	M = 252	164.51
	Readings	from ⊡5,	Elev. = 164.51		
⊡4		2.51	− 0° 45′		
a	208° 02′	2.73	+ 0° 28′	274	166.7
b	218° 53′	1.63	+ 0° 38′	164	166.3
c	228° 02′	0.96	+ 0° 53′	97	166.0
d	277 15′	3.68	+ 1° 59′	369	177.3
e	307° 47′	4.87	+ 1° 04′	488	173.6
f	326° 20′	2.18	+ 1° 02′	219	168.5
g	338° 43′	1.99	+ 0° 48′	200	167.3
h	325° 47′	7.19	− 0° 15′	720	161.4
j	349° 48′	0.71	+ 0° 19′	72	164.9
k	13° 31′	0.72	− 0° 34′	73	163.8



Sta.	Azimuth	Stadia	Vertical Angle	Horiz Distance	Elevation	
	Readings from □5, (cont) Elev. = 164.51					
l	355° 45'	6.02		603		
x-□6	204° 16'	5.53	+ 0° 55'	M = 555	173.38	Should be 173.20
						as per Wilson's Survey
	Readings from □6. Elev. = 173.20					= Original B.M.
□5		5.55	− 0° 55'			
a	159° 12'	1.27	− 0° 14'	128	172.7	
b	172° 20'	2.34	+ 0° 22'	235	174.7	
c	190° 14'	3.22	+ 0° 53'	323	178.2	
d	201° 46'	2.02	+ 1° 12'	203	177.5	
e	207° 24'	1.51	+ 1° 13'	152	176.4	
f	232° 18'	4.52	+ 1° 25'	453	184.4	
g	254° 03'	3.44	+ 1° 18'	345	181.0	
h	256° 45'	3.39	+ 0° 50'	340	178.1	
j	260° 02'	3.29	+ 1° 14'	330	180.3	
k	340° 47'	3.84	+ 0° 04'	385	173.7	
l	10° 14'	1.65	− 1° 07'	166	170.0	
m	13° 11'	2.18	− 1° 07'	219	168.9	
□1	187° 30'	6.27	+ 0° 25'			
	Survey finished May 28th 19–					
	Stadia Readings reduced by Gonzer – June 1st 19–					
	Notes plotted by Griffith – June 1st 19–					



again directed to the rod by means of the lower tangent screw. The upper plate is then unclamped and the telescope is directed to the rod held on the point selected for Station 2. The stadia reading is taken, the vertical angle and the azimuth are read, and the three values, which are, respectively, 4.21, $-1°35'$, and $60°00'$, are recorded in the notes. These readings are made with greater accuracy than those for the side shots.

20. The transit is now set up at Station 2 and oriented in azimuth by taking a backsight on Station 1, with the vernier set at $60°+180°$, or $240°$. The stadia reading and vertical angle from Station 2 to Station 1 are taken as a check; thus, on the backsight, the stadia reading is 4.21 and the vertical angle is $1°34'$. The means between the two stadia readings and between the two vertical angles are generally assumed as the correct readings. Side shots to points adjacent to Station 2 are taken in the same manner as was done at Station 1, as shown in the notes. and Station 3 is then located. By a similar procedure, Stations 4 and 5 and points adjacent to them are located. When the instrument is at Station 3, it is noticed that the point x is plainly visible; therefore, stadia and azimuth readings are taken for that point, and the distance and azimuth thus obtained are used as a check on the plotting of the survey. The last point sighted from Station 5 is the point x at the intersection of Mead and Mill Roads, which was the first point sighted from Station 1 and now becomes Station 6. When the transit is set up and oriented at this station, the azimuth readings on all the instrument points can be checked by sighting to Station 1. If the field work has been done accurately, the reading from x to 1 should be $7°30'+180°=187°30'$.

The horizontal distances and elevations calculated from the stadia readings and vertical angles are included in the notes. In this case, the main purpose of the survey is to locate objects, but the ground elevations at the corners of the buildings and along the banks of the brook are determined and a few contour points are located so that the contours may be drawn roughly by interpolation. However, if the survey had been made primarily for locating contours, the direct method could have been used and side shots taken to points at the desired contour elevations.

TOPOGRAPHY BY PLANE TABLE

21. General Explanation.—When a tract is to be surveyed by the plane-table method, the area is first covered with a control survey as in the case of the transit and stadia method, the stations of this survey being selected so as to obtain the best outlook. These stations and the bench marks with their elevations are plotted on the plane-table sheet or sheets. The plane table is then set up at each station of the control survey, and sights are taken to locate the various topographic features, which are plotted at once on the plane-table sheets. The difference of elevation between each point and the station occupied is determined from the vertical angle and the stadia reading by means of the stadia reduction table or the stadia slide rule. To obtain the elevation of the observed point, the difference of elevation is added to or subtracted from the elevation of the station occupied. This elevation is marked on the map as soon as determined, and the topographer, with the ground before him, is able to determine when enough points have been located to represent the slope of the ground with the degree of accuracy desired for the map. He can also see from the map when all the topographic features and details have been located. Additional plane-table stations may be required for the location of all details on the area.

Points that are desirable locations for instrument stations, but are not a part of the control survey, can be located and the plane-table oriented by the use of the three-point method, which is especially useful when triangulation is used for control. The elevations of these plane-table stations are determined by stadia readings.

The plane-table party is usually composed of a topographer, a computer or a levelman, and two or more rodmen. The topographer makes the observations, plots the points on the map, and sketches the contour lines. The computer makes the necessary calculations for inclined sights. For the direct location of contour points a levelman replaces the computer.

22. Plane-Table Survey.—The method of surveying with the plane table supplemented with a wye level for direct location

of contours is often used and, therefore, will be here described. However, it should be kept in mind that the elevations of points may also be found by stadia and the interpolation method used for plotting contours. In Fig. 3 is represented a portion of a tract of ground on which the contours at intervals of 20 feet are to be located directly. The points A, B, and C have been previously located and their elevations have been determined, as

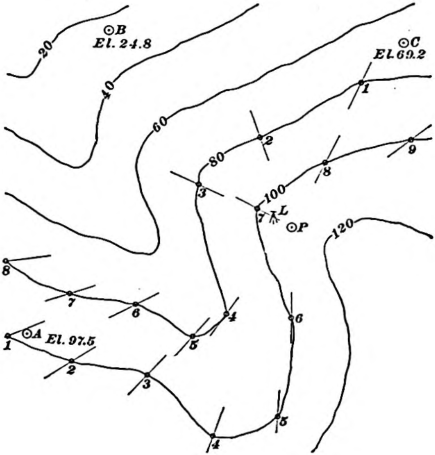


Fig. 3

shown in the figure. The plane table is set up and oriented at a point P, from which is visible as much as possible of the ground to be covered. If the station occupied has not yet been plotted on the map, it is located and plotted before the side shots are taken. It is decided first to locate the 100-foot contour or contour 100; therefore, the level is set up at a point of good outlook such as L, with the telescope of the level somewhat above the contour elevation. In general, the ridge point on a hill is the most advantageous place for a set-up, as it commands the hillside in two directions. A backsight reading of 7.2 is taken

on station A and the height of instrument is thus found to be $97.5+7.2=104.7$. All points in contour 100 are $104.7-100 =4.7$ feet lower than the level line of sight, and the target on the rod is set at 4.7 feet. When the contour interval is so great that points on only one contour can be located from a set-up, it is often convenient to locate the level so that the target is within a foot below the rodman's eyes. He is then able to estimate roughly the elevation by sighting at the level, and at the same time can determine if any leaves or bushes obstruct the line of sight to the target. The manner of tracing the contours on the ground is as described in the following article.

23. The first point 1 of the 100-foot contour in Fig. 3 is determined by moving the rod up or down the slope until the target on the leveling rod, which has been set to read 4.7 feet, is intersected by the line of sight of the level. This point is then located from the plane table and plotted on the map; the line of sight is drawn lightly and the horizontal distance, as determined from the stadia reading and the vertical angle, is laid off on it to the scale of the map, and the elevation 100 is marked lightly beside the point. While the topographer is plotting, the rodman determines about where it is necessary to locate another point to show the path of the contour properly. Here he again moves up or down hill as directed by the levelman until he reaches a point where the target is cut by the line of sight. This point 2 is located from the plane table and plotted, and its elevation marked, in a manner similar to that described for the first point. Points 3, 4, 5, 6, 7, 8, and 9 are plotted in the same manner. From time to time the topographer draws the contour through the plotted points. The order of taking shots is shown in the figure for the purpose of explanation, but is not recorded on the actual map, as it is not important.

After point 9 is located, it is noted by the levelman that his position and that of the plane table will have to be changed to trace the contour further. Such a move is uneconomical until points on higher and lower contours that are within the range of the plane table are located. The leveler therefore sets a temporary turning point by a foresight of 11.2, the elevation of

which is $104.7-11.2=93.5$. Then by a backsight of 1.4 and a foresight of 10.9, another turning point is established whose elevation is $93.5+1.4-10.9=84.0$. The level is set up at a convenient place along the ridge and a backsight of 0.9 gives a height of instrument of $84.0+0.9=84.9$. The target on the rod is then set at 4.9 and points 1, 2, 3, 4, 5, 6, 7, and 8 are located on contour 80. In a similar manner portions of contours 60, 40, and 20 are plotted. Another position is then chosen for the plane table and the contours are extended still farther over the tract.

The farthest point located on each contour in the direction in which the contours are being extended is generally marked with a temporary stake, and a side shot on this point is taken from the next succeeding position of the plane table before the contour is continued. Thus, point 9 on contour 100 and point 8 on contour 80 will each be marked with a stake. It is often advantageous to have a stadia rodman who holds a rod on the contour point that has just been located by the leveler and his rodman. While the topographer is sighting on this rod, the level rodman is selecting the next contour point. If the contour interval is such that the leveler can determine points in two contours from the same setting of the instrument, two rodmen are sometimes employed, one tracing one contour and the other the next one above or below; points on each contour are located and plotted alternately.

24. **Sketching the Contours.**—When several points on a contour are located, the topographer connects them by a continuous line, that in his judgment represents the path of the contour on the ground. Sometimes it is more advantageous to wait until all the points on two or more contours visible from the position of the plane table are plotted before any contours are sketched, because the topographer can then estimate better the proper curvature between points by comparison with adjacent contours.

Sketching is an important part of the topographer's work and a difficult one in which to acquire proficiency. It affords a wide scope for the exercise of judgment and skill, and its accuracy

depends on the ability to see the principal features of the relief and to represent them on the map so that the contours give the correct picture of the surface of the ground.

CROSS-SECTION METHOD

TOPOGRAPHY BY TRANSIT AND LEVEL

25. General Description of Method.—The cross-section method may be used for making the topographical survey of a limited area that is to be laid out as a new town site, an addition to a city, a park, or a cemetery, or that is to be used for any other purpose requiring a knowledge of the topography of the surface.

If the boundaries of the tract have not been previously surveyed, the first step is to run a traverse around the tract, and establish the corners. Then, in order to determine the topography, the tract is usually divided, as far as possible, into squares or parallelograms of uniform size, whose sides may have any dimensions from about 25 to 100 feet or more, as the conditions may require. The form chosen for these divisions will depend somewhat on the form of the tract, and their size will depend on the physical features of the ground and on the degree of accuracy required. The corners of the squares or parallelograms are either defined by stakes or located by ranging out and measuring from stakes already set.

In Fig. 4 is represented a tract of land that is to be surveyed for the purpose of determining the topography of its surface. It is assumed that a traverse survey locating the boundaries has already been made. The tract is to be divided by means of lines in two perpendicular directions, and 100 feet apart. In dividing tracts in this manner, it is customary to designate by letters the lines that extend in one direction, and by figures the lines at right angles to that direction. The point at the intersection of any two lines is then designated by the letter and figure of the respective intersecting lines. Thus, the lines perpendicular to the side selected for a base are designated as lines *A*, *B*, *C*, etc., while the lines parallel to the base are designated as lines *1*, *2*, *3*, etc. The intersection of the line *D* and the line *1* is designated

as *D1*, the intersection of the line *E* and the line *5* is designated as *E5*, etc. The intersections of the dividing lines with the other boundary lines are designated by the proper letters or numbers affected with an accent or a subscript, as A', B', 2_1, 3_1, etc.

26. Staking Out the Tract.—Different methods may be followed in laying out the tract. Any method is satisfactory that accurately defines the positions of the points of intersection, so that they can be readily located when the levels are being

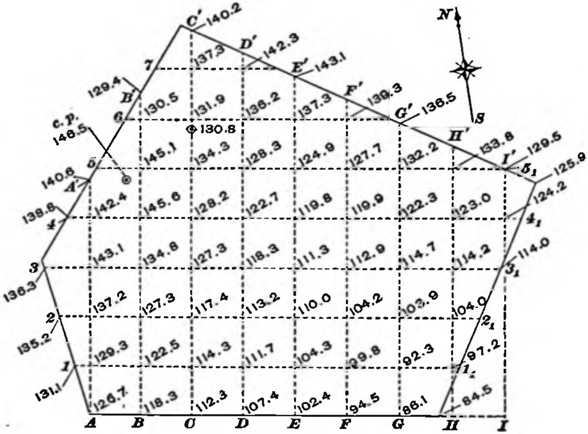


Fig. 4

taken. For this purpose it is not usually necessary to mark all points of intersection, but a sufficient number of points should be marked by stakes so that the remaining points can be located easily and quickly by merely ranging them in from the points that are marked. The rougher and more irregular the surface of a tract, the more stakes must be set, and a tract of irregular form usually requires a comparatively greater number of stakes than a tract of rectangular form. If the tract is rectangular in form and its boundary lines are complete and unbroken, so that the stakes can be set on each boundary line along the entire

length of the side, and if the tract is comparatively level, so that the stakes on one boundary line can be seen from the corresponding stakes on the opposite boundary line, it will not usually be necessary to set stakes at the points in the interior of the tract. For, in taking the levels over such a tract, the leveling rod can be ranged in between the stakes on the boundary lines.

The surface of the tract represented in Fig. 4 is somewhat irregular, as shown by the elevations written at the intersections of the lines. For these conditions, the following method may be adopted in staking out the tract: The south side is taken as a base line, and stakes are set along this line at intervals of 100 feet, these points being marked *A, B, C,* etc. At each of these points, a line is run across the tract at right angles to the base line, stakes being set at intervals of 100 feet and at the intersections with the boundary lines. The base line is extended beyond the tract so that line *HH'* may be run.

No cross lines are run in the field parallel to the base line, but the squares are completed in the sketches and on the map by drawing lines through *A1* and *H1*, through *B2* and *H2*, etc. Moreover, where necessary to complete the topography, such cross lines are extended to the boundaries of the tract by ranging in with stakes previously set. Thus, after the stakes at *A1* and *B1* have been set, the stake at *1* is put in line with them and on the boundary line. Likewise, the stake at *2* is put in line with *A2* and *B2* and so on. Where such intersections are close to other points previously established on the boundary, as at *A'*, which is close to *5*, and at 1_1, which is close to *H1*, no stakes need be set. The stakes are placed merely for elevations and there is no necessity for two readings very near each other. The point *I4* is the only intersection on line *I* and is ranged in from *G4* and *H4*, the distance from *H4* being taped or paced as desired. In this case, the points 3_1 and 5_1 can be located more readily than the intersections of the line *I* with the boundaries. Care should be taken to have the stakes, as shown on the sketch in the notebook, numbered in the same manner as on the ground.

27. Taking the Levels.—After the required stakes have been set, the levels are taken over the tract, determining the

elevations at all points of intersection and at any intermediate points where the slope changes abruptly. Such an intermediate point is generally located in a direct line between two intersections by its distance from the intersection having the lower letter or number. This distance is measured with a tape, approximated by pacing, or merely estimated by the eye, according to the conditions and to the degree of accuracy required, and is recorded as a plus. Thus, on line *CC′* there is a low point 80 feet beyond stake *C5* and its elevation is 130.8; this point would be designated as *C 5+80*. The tops of knolls and the low points of depressions are generally located when they are not crossed by the line between two intersections. The high point marked *c.p.* (contour point) whose elevation is 146.5 and which is situated between lines *4* and *5*, and between lines *A* and *B*, is such a point; it would be designated in the notes as *A+80, 4+75*.

The levels should be taken in the order that is most advantageous for the nature of the ground, the object being to take rod readings at each of the intersections and at other points with as few settings of the level as possible. To be sure that rod readings are taken at all the intersections, those taken from each setting are checked off on the sketch or sketches in which they are all shown.

28. **Form of Notes.**—The notes are substantially the same as ordinary level notes, with the addition of sketches on the right-hand page, as in the stadia method, showing the form and dimensions of the tract surveyed, the manner in which it is divided, and the method of designating the points. Each point is designated by its letter and number, since the levels are not usually taken successively along one line.

TOPOGRAPHY BY HAND LEVEL

29. **Hand Level.**—In Fig. 5 is illustrated a typical hand level, which is also called the Locke level from the name of the inventor. It consists of a brass tube *AB* about 6 inches long—which is usually finished in bronze or nickel-plated—that has on the top near the object end a small spirit level *C*. Beneath the

level is an opening in the tube through which, by means of a reflecting prism placed below it, the bubble can be seen when the eye is placed at the small opening in the eye end *D*. The reflecting prism occupies one-half the cross-section of the tube and is set at an angle of 45° to the line of sight. Any object toward which the instrument is directed can be seen through the part of the tube that is unobstructed by the prism. A cross-wire is placed directly below the center of the level in such position that, as reflected, it will bisect the bubble when the line of sight is horizontal. If the tube is held to the eye and directed toward an object, and its object end is raised or lowered until the bubble is bisected by the cross-wire, the point on the observed object with which the cross-wire coincides, as seen through the unobstructed portion of the tube, is at the same elevation as the eye.

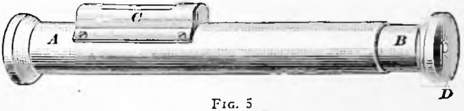


FIG. 5

30. Use of Hand Level.—The hand level is used for locating contours where a narrow strip of topography adjacent to route lines is desired, such as the topography on either side of the traverse line for a railroad or highway. The traverse is first run with stakes placed at each 100-foot station, and the ground elevation at each station is established with a wye level. The cross-sectioning may then be done by taking levels along lines at right angles to the traverse, each cross-section being extended as far as deemed necessary. A topographer or an observer and one chainman with a rod can do the work, but it can be carried on more rapidly if there are two chainmen.

As the support for the hand level consists generally of a 5-foot stick against which the level is held, it is not very steady and, hence, the rod viewed through the level is not magnified. For this reason, the length of sight is limited to the visibility of rod readings with the naked eye. Where the target is used, the length of shot may be increased somewhat. The equipment for taking cross-sections with a hand level consists of a field note-

book, with the right-hand page cross-ruled in $\frac{1}{4}$-inch squares, a hand level, a 5-foot stick, a self-reading leveling rod, and a 100-foot tape.

31. **Direct Location With a Hand Level.**—The following example serves to explain the cross-section method of locating contours directly with a hand level, a contour interval of 5 feet being used. The notes for a portion of a traverse line between Stations 105 and 109 are recorded in Fig. 6. On the left-hand page of the notebook are entered the station numbers and the elevations of the ground at the stations on the traverse line. On the right-hand page are made sketches showing the locations of the contours to scale. The heavy line at the center of this page is red in the notebook and represents the traverse line. The small spaces are $\frac{1}{4}$ inch square and the scale used for the sketch is 1 inch = 100 feet. The notes should begin at the bottom of the page so that, when the topographer faces in the direction in which the station numbers increase, objects on either side of the line can be sketched in their natural positions with reference to the traverse line.

32. In Fig. 7 is represented the cross-section at Station 105. To take the measurements at this station, the leveler first holds the 5-foot stick, with the hand level against it, on the ground at the station, which is marked A in the figure. Since the elevation of the ground at A is 958.7 feet, the hand level is at an elevation of $958.7+5=963.7$ and, to locate contour 955, the target on the leveling rod is set to read $963.7-955=8.7$ feet. The rodman moves the leveling rod along the downhill slope on a line at right angles to the traverse line until the line of sight of the hand level cuts the target. Then the rod is on the 955-foot contour, marked B in the figure. This point is located by measuring the distance from A, which is found to be 138 feet. The topographer plots the point B on the cross-ruled, right-hand page of the notebook as shown in Fig. 6, and marks the distance from the traverse line to point B.

The leveler now moves to point B, Fig. 7, and holds the hand level on the 5-foot stick at this point. The elevation of the level is $955+5=960$ feet, and to locate the 950-foot contour the rod-

Station	Elevation
109	956.8
108	950.7
107	938.4
106	948.6
105	958.7

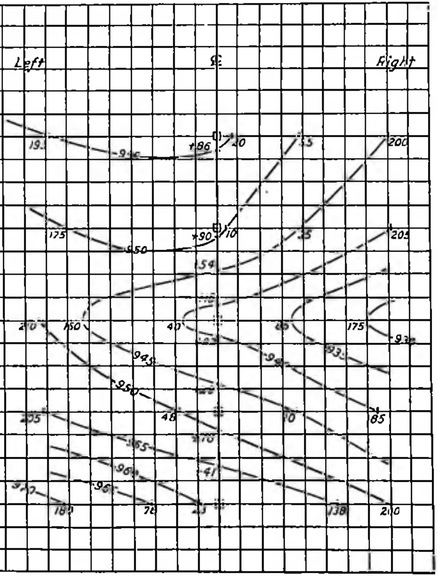


FIG. 6

man moves down the slope until a point is found where the rod reading is 960−950=10 feet. This point is marked *C* in the illustration. The measured distance from *B* to *C* is 62 feet, making the total distance from the traverse line to *C* 200 feet. The topographer plots the point in the notebook, and marks its distance from the traverse line. If the cross-sections are to be carried about 200 feet on each side of the line, no further points are necessary on this side.

33. The contour points on the left-hand side of the station *A*, Fig. 7, are next determined. The leveler again holds the hand level on its 5-foot stick at the station. Since the elevation of the hand level is 963.7 feet, the rodman moves up the slope at right angles to the center line until a point is found where the rod reading is 3.7 feet and thus locates the 960-foot contour, marked *D*. The distance to *D* from *A* is found to be 23 feet. This point is plotted on the left-hand side of the traverse line on the cross-ruled page of the notebook, as shown in Fig. 6. Since the ground slopes up from the traverse line on this side, the leveler goes ahead with the hand level and the rodman holds the leveling rod at point *D*, Fig. 7. The leveler now moves up the slope to a point where, with the hand level held on the 5-foot stick, he makes a reading on the leveling rod of 10 feet. This point, which is marked *E*, is on the 965-foot contour. The distance from *D* to *E* measures 55 feet, and the topographer plots the point, at a distance of 55+23=78 feet from the traverse line, on the cross-ruled page of the notebook. The rodman now moves up and holds the leveling rod at the point *E*, while the leveler goes ahead with the hand level and locates the point *F* on the 970-foot contour, in the same manner as point *E* was located from point *D*. In Fig. 7 the full line shows the surface plotted from the cross-section notes, and the dotted line shows the true ground surface.

This procedure is repeated at every station along the line. Points where the contours cross the survey line are also located by plus distances from the preceding stations. In case the ground along a portion of the cross-section is so flat that the 5-foot contours are too far apart for a sight, the elevations of

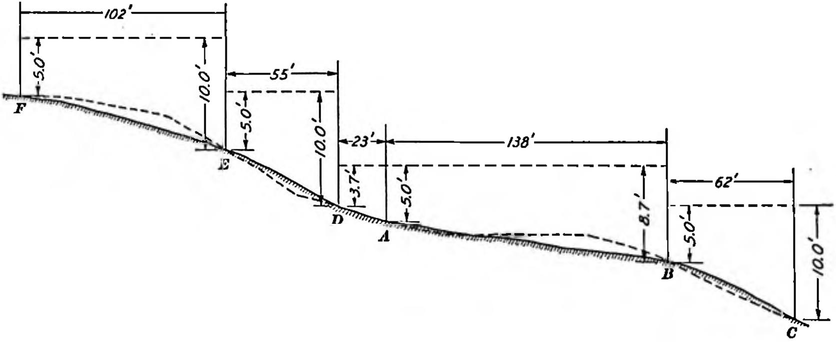


FIG. 7

points at regular intervals, such as every 50 feet, should be determined, until the next contour or the required width of the section is reached.

The contour lines are drawn through the points of equal elevation as shown in Fig. 6. As soon as a point is plotted in the notebook, the contour line on which it lies is started and the elevation of the contour is marked on the line. When a point at a succeeding station is located in the sketch, the contour having the same elevation is immediately extended to pass through that point, the contour lines being adjusted and smoothed out as the plotting progresses. With the ground before him, an experienced topographer can usually draw the contours quite readily in this manner, and it is sufficient to mark the elevations along the contours, as in Fig. 6. However, where there is doubt concerning the path of a contour, it may be desirable to plot a few points before the contour lines are drawn; in this case, the elevations of the plotted points, as well as their distances from the traverse line, should be recorded. The contour elevation is then written above each point, and the distance below it.

34. Interpolation Method With Hand Level.—Sometimes, instead of locating the 5-foot contours on the cross-sections, elevations of points at the breaks in the slope along the cross-section lines, and their distances from the center line, are determined. The contours are then located by interpolating between these points. This method of interpolation saves a great deal of time in the field when the slopes are fairly steep or exceptionally flat. Thus, a constant slope would require only one sight at the top and one at the bottom, even though it covered a vertical distance of more than 10 feet. Also, a cross-section using this method gives a true ground surface. The cross-section at traverse station 105, Fig. 6, is shown in Fig. 8, where the full line represents the ground surface as plotted by the interpolation method from notes recorded as in Fig. 9. In order to show a comparison of this cross-section with that obtained by the method of direct location of contours, the section established from the notes in Fig. 6 is also reproduced in Fig. 8, being represented by the dotted line *CBADEF*.

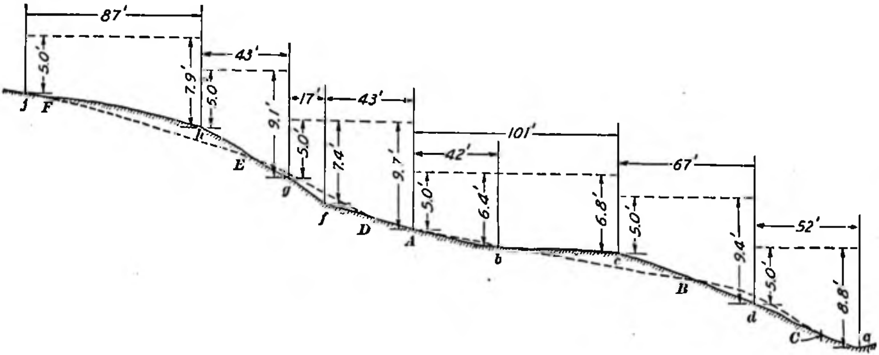

87'
43'
17'
43'
101'
42'
67'
52'
5.0'
7.9'
5.0'
9.1'
5.0'
7.4'
9.7'
5.0'
6.4'
6.8'
5.0'
9.4'
5.0'
8.8'
j
F
E
g
f
D
A
b
c
B
d
C
a

FIG. 8

Station	Elevation	Left	℄	Right
109	956.8	954.9/204 955.2/163 956.7/85 957.2/35		95[illegible].3/65 944.6/210
108	950.7	949.3/204 951.8/112 952.[illegible]/65		948.0/43 943.2/127 940.0/205
107	938.4	947.2/91 94[illegible].1/13[illegible]		936.9/43 929.7/19[illegible]
106	948.6	956.[illegible]/212 953.[illegible]/115 949.0/25		946.[illegible]/50 942.1/130 939.4/190
105	958.7	970.4/90 967.5/10[illegible] 963.4/60 961.0/43		957.3/42 956.9/101 952.5/168 940.7/22[illegible]

Fig. 9

35. In locating the contours by interpolation, the rodman holds the leveling rod at point *b*, Fig. 8, which is the first break in the surface slope to the right of the traverse line, and the leveler at *A* holds the level on the 5-foot stick as in direct location and takes the reading of the rod, which is 6.4 feet. The horizontal distance from *A* to *b* is measured and found to be 42 feet. Since the ground elevation at *A* is 958.7 feet, that at *b* is $958.7+5.0-6.4=957.3$ feet. This elevation and the distance 42 feet are recorded in the notes just to the right of the traverse line, as shown in Fig. 9. The rodman next moves to point *c*, where the rod reading is 6.8 feet, the distance from *A* is 101 feet, and the elevation is $958.7+5.0-6.8=956.9$ feet. It is probable that beyond this point the reading of the rod from *A* will become too difficult. Therefore, a turning point will be taken at *c*, even though the rod is sufficiently long to be read at *d*. The leveler moves up to *c* and reads the rod at point *d*, and the distance *cd* is measured. He then moves to point *d* and takes the rod reading at *e*, and the distance *de* is also determined. The recorder notes all distances as measured from the traverse line. Thus, for the point *d*, which was found to be 67 feet from *c*, the distance recorded in the notes is $101+67=168$ feet.

36. After point *e*, Fig. 8, has been located, the same process is carried out on the left side of *A* with the leveler moving first to point *g* and taking readings at *A* and *f*. Also, the distances from *A* and *f* to *g* are measured. The rod reading at *A* from point *g* is found to be 9.7 and at *f* is 7.4. Hence, the elevation of *f* is $958.7+9.7-7.4=961.0$ feet and the elevation at *g* is $958.7+9.7-5.0=963.4$ feet. The rodman then moves up to *g* and the leveler goes to the next break in the surface slope, which is at *h*. The rod reading is taken and the distance measured. The leveler then moves to point *j* and the rodman to point *h* for the final reading and measurement on the section. The elevations and distances are recorded in the notes as in Fig. 9.

It may be seen that where an actual cross-section is desired this method is superior, but when a map is to be made and contours are the important factor, the direct method is generally better suited for locating true contour elevations.

SLOPE-LEVEL METHOD

37. The Rate of Slope.—The rate of slope may be determined either by measuring the horizontal and vertical distances between two points in the slope or by measuring the angle of the slope. In the first method, the vertical distance between some point on the slope and the point at the beginning of the slope is measured with a level and a rod, and the horizontal distance between those points is measured with a tape. A hand level is generally used in this work.

Slopes of the natural surface are usually designated by the vertical rise or fall in the corresponding horizontal distance. Thus, a slope of 1 in 20 indicates a rise or fall of 1 foot in 20 feet measured horizontally. In a cross-section of an excavation or an embankment for a railroad or a highway, a slope is generally designated by the ratio of the horizontal measurement to the vertical measurement, the latter being reduced to 1. For instance, a slope of 20 feet vertical in 30 feet horizontal is designated as a $1\frac{1}{2}$ to 1 slope, and is usually expressed as $1\frac{1}{2}:1$.

Slopes are sometimes designated by the slope angle. Thus, a 45° slope indicates an angle of 45° with the horizontal. The slope angle may be measured directly by means of an instrument called a *clinometer*.

38. The Clinometer.—A simple form of clinometer, or slope level, is shown in Fig. 10; it consists of a straight bar *ab* about 6 inches long, to which is attached the movable arm *mn* that carries a spirit level *l* and turns on a hinge at *m*. The direction of the arm with reference to the bar *ab* is shown by the quadrant scale *qr*, which is graduated in degrees. If the bar *ab* is placed on any sloping surface and the arm *mn* is raised until the bubble is at the center of the level tube, the arm will be horizontal and its reading on the graduated quadrant will be the angle that the slope makes with the horizontal. Since the bar *ab* is short and the surface of the ground is usually uneven, a board about 10 feet long, called a *slope board*, or *slope rod*, is generally used with the clinometer in order that the measured slope of the surface of the ground shall be its average slope. This board has one straight edge with a portion of the opposite edge parallel.

The straight edge is placed on the sloping surface of the ground and the bar *ab* of the clinometer is placed on the opposite parallel edge of the board. It is sometimes found convenient to attach the clinometer permanently to the slope board.

39. **Abney Level and Clinometer.**—As now commonly made, the clinometer is combined with the hand level, and the combined instrument is known as the Abney level and clinometer. Such an instrument is shown in Fig. 11. The main tube *a* is similar in construction to the tube of an ordinary hand level, but the small spirit level *b* on top of the main tube is movable

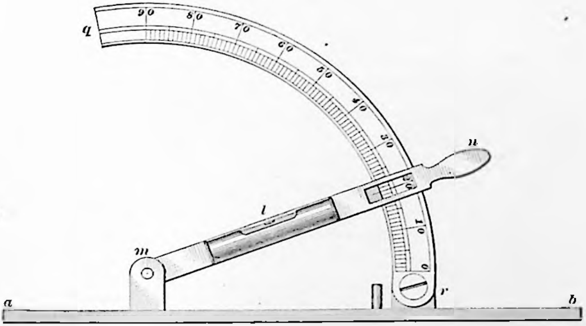


FIG. 10

in a vertical plane. When the main tube is given any inclination, the level can be turned to a horizontal position. as indicated by the bubble, and the angle of inclination of the tube with the horizontal is shown by means of a graduated arc *c*, attached to the instrument, and the vernier *d*. When using the instrument, the observer can look through the eyepiece *e* and, while holding the main tube *a* at the desired inclination, can adjust the position of the level *b* so that the reflected image of the bubble is bisected by that of the cross-wire. The level tube is then horizontal and the reading of the arc is the required angle. Also, since the main tube is square in cross-section and its lower side is straight, it can be laid on any surface and the instrument used as an ordi-

nary clinometer. When the spirit level is set in a position parallel to the main tube so that the reading on the graduated arc is zero, the instrument can be used as a hand level.

The graduations on the inner edge of the arc at *f* indicate the slope ratio, the index being at *g* on the same arm as the vernier *d*. The outside graduation on each side indicates a slope of 1 to 1, the next $1\frac{1}{4}$ to 1, etc., up to 10 to 1 at the inside.

40. Cross-Sectioning With the Clinometer.—The clinometer is sometimes used instead of the hand level in cross-sectioning. In this method, the inclination of the slope of the ground is determined with the clinometer and the distances are

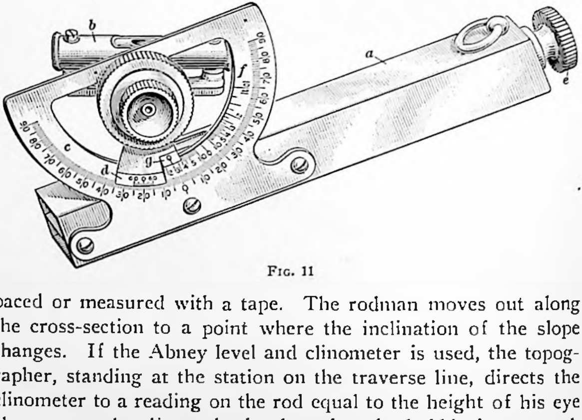


FIG. 11

paced or measured with a tape. The rodman moves out along the cross-section to a point where the inclination of the slope changes. If the Abney level and clinometer is used, the topographer, standing at the station on the traverse line, directs the clinometer to a reading on the rod equal to the height of his eye above ground, adjusts the level so that the bubble is centered, and reads the angle of inclination. The distance to the point is measured, and from the distance and the angle of inclination the difference of elevation between the two points is determined. This difference is equal to the product of the horizontal distance and the tangent of the angle. Thus, if the horizontal distance is 100 feet and the slope angle is 5°, the difference of elevation is 100 tan 5°, or $100 \times .087 = 8.7$ feet. For convenient use in the field, there are tables from which the difference of elevation can

be taken directly. By adding this difference to the elevation of the station if the ground slopes up from the traverse line, or subtracting it from that elevation if the ground slopes down from the traverse line, the elevation of the point where the leveling rod was held is obtained. If the ground elevation at the station is 164.8 feet, the ground slopes up from the traverse line, and the difference in elevation is 8.7 feet, as just assumed, then the elevation of the point at which the rod was held is $164.8+8.7=173.5$ feet. This point is plotted and its elevation 173.5 and the distance 100 from the traverse line are entered in the notebook, as in cross-sectioning with a hand level.

Next, the topographer moves to the point previously occupied by the rodman while the latter proceeds to the next break in the slope. The distance between these two points is measured, the slope angle is determined by means of the clinometer, the difference in elevation between the points is calculated, and the distance from the traverse line to the rod and the elevation of the ground at the rod are found and recorded in the notebook. If, for the cross-section previously considered, the distance from the topographer to the rodman is 83 feet and the slope angle is $3°15'$, then the difference in elevation between the two breaks in the ground is $83 \tan 3°15'=4.7$ feet; the distance from the traverse line to the new point is $100+83=183$ feet, and the elevation of that point is $173.5+4.7=178.2$ feet. The elevations and distances from the traverse line are found in this way for all points on the cross-section where the inclination of the ground changes, and are entered in the notebook.

If a distance is measured along the slope, the corresponding horizontal distance is equal to the product of the slope distance and the cosine of the angle of inclination; also, the difference of elevation is equal to the product of the slope distance and the sine of the angle of inclination.

AERIAL TOPOGRAPHIC SURVEYING

PHOTOGRAPHS FOR TOPOGRAPHY

41. Aerial Photography in Surveying.—With the aid of very sensitive instruments it is possible to apply aerial photography, or photographs taken from airplanes, to the making of practical topographic maps. The principal uses of aerial photography have been in reconnaissance work for the location of high-tension power transmission lines, pipe lines, railroads, and reservoir and dam sites; and for government surveys, timber cruising, park and landscape gardening, and city planning and traffic studies. It is now being used to furnish accurate finished topographic maps of large areas for various engineering projects. Such work can be carried out only by organizations outfitted with highly specialized equipment for this purpose.

42. Methods of Producing Aerial Photographic Maps. There are two general types of aerial photographs used in topographic work, each type dependent on the location of the camera at the time the picture is taken. In one type, which results in the common bird's-eye view or *oblique photograph,* the camera is held with its axis at an angle with the vertical. In the other type, the camera is pointed vertically downwards and the developed pictures, known as vertical-axis pictures, are placed together or matched to form what is called a *mosaic map.* However, distances cannot be scaled with accuracy from such maps. A specially prepared mosaic map, in which the position of each picture is carefully oriented and the scale standardized, is called a *photographic map.* This type of uniform-scale map is replacing the less accurate mosaic map, except in very rough, temporary work.

43. Contour Maps From Aerial Photographs.—In nearly all aerial survey work, use is made of the *stereoscope* or the stereoscopic principle. The human eyes are an example of the stereoscope. Each eye sees an object from a slightly different angle and thus gives a three-dimensional image to the brain. By mechanical apparatus the same effect is brought about in aerial photographic surveying.

It has been found impractical to attempt to prepare aerial topographic maps without reference to some ground control, both as regards elevations and distances. The amount of ground work depends on the method used for placing the contours on the finished map. Three general methods now in use are as follows:

(1) In the method used by the United States Geological Survey, which is especially adapted to the production of maps on small scales such as 1 : 62,500, either vertical-axis or oblique photographs are taken from the air. In the latter case, the oblique views must be rephotographed so as to conform to vertical-axis pictures. Then the principal topographic features are transferred onto plane-table sheets by means of suitable reduction processes, and the contours are located on these sheets by topographers in the field in the usual manner. The aerial photographs are thus employed as aids or guides in plotting the topography.

(2) In another method, which is suitable for large-scale maps such as are required in connection with engineering projects, the true adjusted photographs or enlargements of them are used as plane-table sheets by the ground crew, and the contours are sketched in the field directly on the original sheets. The resulting map may be reproduced by photographic methods or traced for commercial use.

(3) The method known as *stereophoto surveying* is more recent, and requires special patented machinery for complete, accurate results. The only ground work necessary for this type of survey is to establish a base line of known length at each end of a short survey or at suitable intervals in a long survey, and to determine the relative elevations of four points on each photograph. By mechanical corrections, reprojections, and stereoscopic views, it is possible to draw contours directly on the photographic map without further field work.

44. Advantages and Disadvantages of Aerial Surveys. Topographic maps prepared in conjunction with aerial photographs show the details more completely than maps based on the usual ground surveys alone. For large surveys, the aerial methods are also cheaper and quicker. From an aerial reconnaissance

map, it is possible to determine immediately the desired extent and need for accurate surveys without sending out a ground party for a preliminary survey. In heavily wooded areas, where ground surveys are difficult, the aerial survey method is especially advantageous, because it avoids the clearing of trees and bushes and the establishing of numerous base camps.

For small areas or flat open ground, where one-foot contour intervals are desired, aerial methods are too expensive and not sufficiently accurate. The charges for photographic work and the cost of flying far exceed the expense of a ground crew in small surveys. Accurate ground controls are always necessary in aerial work, which means precise leveling throughout the area as well as an accurate base-line measurement at the end of each survey and at intervals of about 15 miles in long surveys.

HYDROGRAPHIC SURVEYING

Serial 787 Edition 2

SURVEY OF OUTLINE OF A BODY OF WATER

1. Definition and Object of Hydrographic Surveying.—Hydrographic surveying comprises all surveys of lakes, streams, reservoirs, or other bodies of water. Its object may be:

1. To obtain sufficient information from which to draw an outline map of the body of water surveyed.

2. To determine, in addition, the elevations of a sufficient number of points on the bottom below the water surface to define the subaqueous contours of the containing valley or basin.

3. To determine the form of a portion of the bottom of the sea, a bay, harbor, or navigable river, for purposes of navigation. In this case, it is necessary to locate the navigable channels, and the obstructions to navigation, such as shoals, rocks, sunken wrecks, etc.

2. Limitations.—Hydrographic surveying consists merely in making the measurements necessary for acquiring such information as is outlined above. The measurement of the velocity and discharge of rivers and streams, and the planning and execution of works of improvement, such as the reclamation of submerged areas or the construction of breakwaters, sea walls, dams, etc., belong to hydraulic engineering, and will not be treated here.

3. Traverse Survey.—The survey of a body of water to determine its outline may be conducted as an ordinary traverse by any of the methods described in *Transit Surveying.*

The courses of the traverse are run at convenient distances from the water's edge and the shore line is determined by measurement from the line of the survey. The position of the ordinary low-water line is usually defined, but in many cases the high-water line is also determined and noted.

A good way to make an outline survey of a body of water is by means of a deflection traverse, using a transit and a

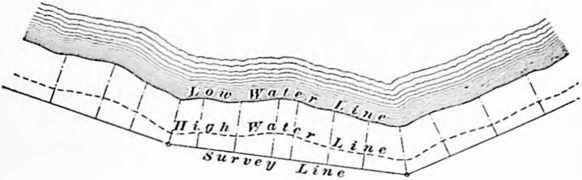


Fig. 1

chain or tape. This method is commonly used and is satisfactory for ordinary surveys of this kind. The outline survey of a body of water can also be made by the transit and stadia; this method of surveying is fully described in *Stadia and Plane-Table Surveying.* The entire survey of a small

Fig. 2

river or stream, including the location of soundings, can be made with the stadia.

Prominent objects on shore may be located by direct measurement from the line of survey. If at a considerable distance, they may be located by triangulation, by taking sights from two known points. The distances from the line of the survey to the high- and low-water lines are usually measured by offsets, as illustrated in Fig. 1. If it is not necessary to obtain a close approximation of the shore outline,

the offset measurements can be omitted and the shore line between the survey stations sketched in by the eye.

In the case of a small stream, a traverse run along one bank is usually sufficient. The offsets should be measured to the edge of the water, and the width of the stream also measured at sufficiently close intervals to give the required information, as shown in Fig. 2. In the case of a small lake, the traverse is run entirely around it and closed on the point of beginning.

4. **Triangulation.**—A triangulation survey probably affords the best means for determining the outlines of large rivers, lakes, and other large bodies of water. Triangulation, as applied to hydrographic surveying, consists: (1) in locating distant objects from a measured base; (2) in determining the surface outlines of a river or other body of water by a system of triangles referred to a measured base.

The base line should be measured on fairly level ground in a location convenient for making the angular measurements from its ends. It should be not less than 500 feet long and as much longer as practicable. The ends should be marked with substantial stakes or with stone monuments. The line should be measured carefully with a steel tape.

5. **Locating Distant Objects.**—For locating points of reference and other distant objects, the angle formed by the intersection of the base line and the line of sight to the object, at each end of the base line, is measured. From the values of these angles and the length and azimuth of the base line, the lengths and azimuths of the lines to the object can be calculated and the object located. Let AB, Fig. 3, represent a base line and C a distant object whose position is to be determined. The base line AB is measured accurately, and the angles ABC and BAC are measured with a transit or

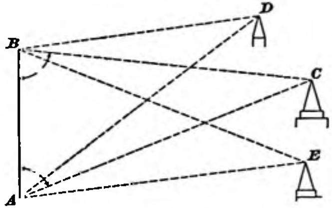


FIG. 3

sextant. Then, in the triangle $A\,B\,C$, the side $A\,B$ and the adjacent angles $A\,B\,C$ and $B\,A\,C$ are known, from which the sides $A\,C$ and $B\,C$ can be calculated by trigonometry, and the point C located. The points D and E can be located in a similar manner from the same base line.

6. Triangulation of River.—For the survey of a river or other body of water by triangulation, points are selected on both sides for the vertexes of the triangles. Such points are called **triangulation stations.** They should be so located as to give triangles of advantageous form, in which no angle will be less than 30° or greater than 120°. Fig. 4 illustrates the triangulation of a river for the purpose of determining the outline of its shores. Some convenient line, as the line *1–3*, is taken as the base, its length is carefully measured, and its azimuth is either determined or assumed. The angles from *1* and *3* to *2*, and the angle *2*, are carefully measured. Their sum should not differ from 180° by more than 1 minute. The difference between that sum and 180° is distributed equally among the three angles of the triangle, one-third of it being added to, or subtracted from, each angle, as may be necessary to bring the sum to 180°. The same applies to the other triangles, in each of which each angle should be measured directly.

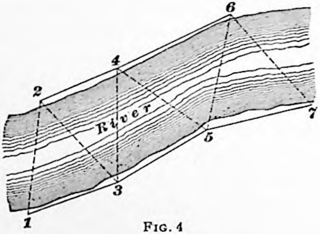


FIG. 4

Knowing the angles of the triangle *1–2–3*, and the length of *1–3*, the lengths of *1–2* and *3–2* are computed by trigonometry. Then, in the triangle *2–3–4*, the angles and the side *2–3* are known, and the other two sides are computed; and so on with the other triangles. At the end of the chain of triangles another line, as *5–7*, whose length has been calculated, is measured, as a check on the work.

The shore line between triangulation stations can be sketched in approximately, or, if it is desired to determine its outline more closely, the more important points can be located with transit and stadia, or by intersections from two triangulation stations when these are so situated as to give satisfactory intersections. If more detailed information is desired, a traverse can be run between adjacent stations and the shore line located by offsets at such intervals as may be desired.

SURVEY OF A SUBMERGED AREA

7. Purpose of Survey.—A hydrographic survey to determine the topography of the bottom of the basin or channel containing a body of water may be made for one or more of the following purposes:

1. To determine what changes it is desirable or necessary to make in the configuration of the channel or basin under consideration.
2. To indicate where material should be removed by dredging or blasting and where it may be deposited for filling, and to measure the quantity of material removed or the extent of the filling.
3. To obtain the information necessary for planning the construction of sea walls, jetties, lighthouses, docks, bridge piers, etc.
4. To construct a map or chart of the channel or basin for navigation purposes.
5. To determine the volume of the body of water, or capacity of the containing basin.

In making the survey of a submerged area, it is first necessary to make an outline survey in order to determine the shore line and locate points of reference. The points of reference are usually on shore and may be located by direct measurement or triangulation, as may be more expedient. In some cases, buoys are anchored in the water and used for reference points; as they are inaccessible, their distances from other points of reference must be determined by computation.

FIG. 5

SOUNDING

8. Soundings.—The shore line having been determined and the reference points located, the next step is to measure the depths, below the water surface, of a sufficient number of points to show the configuration of the bottom; such measurements are called **soundings.** For depths of 18 feet or less, soundings are made with a graduated wooden rod called a **sounding pole.** For greater depths, a line having a weight attached is necessary; this is called a **lead line.**

9. Sounding Pole.—The sounding pole may be of any sound, straight-grained wood. It should be well seasoned to prevent warping, and the bottom end should be provided with a disk-shaped iron shoe, not less than 5 inches in diameter, to prevent the rod from sinking into the soft mud of the bottom.

A good form of sounding pole is illustrated in Fig. 5. Poles of this kind are usually made of white pine finished smooth and round. The length is usually from 15 to 20 feet, and the diameter from 3 to $3\frac{1}{2}$ inches at the lower end and from 2 to $2\frac{1}{2}$ inches at the upper end. The lower end of the pole is formed by an iron shoe that terminates in a disk, as shown.

The pole is painted white and is graduated to feet and tenths, the zero of the graduation being at the bottom of the shoe. Each foot division is marked by a red band about $\frac{1}{8}$ inch wide, and each tenth division by a black band about $\frac{3}{16}$ inch wide; the bands extend entirely around the pole. The graduations are numbered by two sets of figures placed on the pole diametrically opposite each other. The

numbers designating feet are painted in red and those designating tenths in black.

The bottom of the shoe is sometimes hollowed out cup-shaped for the purpose of bringing up samples of the bottom over which the soundings are taken. When samples of the bottom are desired, the cavity is lined with grease or tallow, to which particles of the sand or mud of the bottom will adhere.

10. Lead.—The weight attached to a sounding line is called the **lead**, because it is usually made of that material. It should be long and slender, and should taper slightly toward the upper end, so as to reduce its resistance to being raised in the water. The form shown in Fig. 6 is frequently used. An iron rod R has molded around it the lead L, which is usually square in cross-section, as shown at S, and of sufficient size to give the requisite weight. Small cross-bars attached to the rod prevent the lead from slipping. At the lower end of the rod is attached the cup C, which is covered with a leather washer W that slides freely on the rod between the cup and the lead. When the lead is lowered to the bottom, the cup sinks far enough into the bottom to fill, and the leather cover prevents the contents from being washed out while the lead is being drawn to the surface. In some cases the cup is omitted and the bottom of the lead hollowed out in conical form. When it is not desired to know the composition of the bottom, a plain lead of nearly cylindrical form, but tapering toward the upper end, will answer the purpose. For still, shallow water, a lead weighing about 5 pounds is satisfactory. A lead weighing 10 pounds is suitable for depths under 40 feet in reasonably quiet water. For greater depths and in strong currents the weight of the lead should be from 15 to 20 pounds.

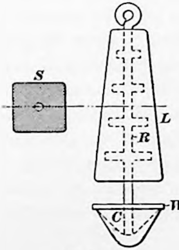


FIG. 6

11. Sounding Line.—Preferably, the sounding line should be of strong, closely plaited linen or twisted hemp.

Sometimes a cotton rope or a wire chain is used, but the use of such materials is not recommended. The line should be of a size suited to the weight of the lead; for ordinary river or lake soundings, about $\frac{3}{8}$ inch in diameter is a good size. It is marked with leather or cloth tags, which are inserted between the strands of the line. For river and harbor surveys, the tags are placed at intervals of 1 foot. At every fifth or tenth interval a conspicuous tag, usually of a bright color, is used. The zero of the graduation is the bottom of the lead.

Before being measured and marked, the line should be thoroughly stretched. This is done by stretching it tightly between two posts or trees, or wrapping it closely around a post or smooth-barked tree, then fastening both ends, wetting thoroughly and allowing it to dry. The slack is then taken up and the operation repeated until the line shows no further slack. Care should be taken not to stretch it too much, as in that case it will shorten in use. The length of the lead line, from the end of the lead to each 10-foot mark, should be tested before and after each day's use, and the results entered in the notebook. The line should preferably be kept under water when not in use; if it is not, it should be soaked in water for $\frac{1}{2}$ hour and then tested for length before the sounding is commenced.

12. Sounding Party and Equipment.—When the soundings are located by means of observations made with instruments stationed on the shore, the sounding party may consist of the recorder, leadsman, and boat crew. If the soundings are located by the stadia method, a stadia rodman is added to the sounding party. When the soundings are located from the boat, the sounding party is usually composed of two observers, a recorder, a leadsman, and the boat crew.

The usual equipment of a sounding party consists of a sounding pole or lead line and two signal flags, one white and one red. The flags are used to signal to the instrumentman on shore when a sounding is being taken, if the soundings are located by an instrument on shore. The white flag

is shown for each sounding except every fifth one, when the red flag is shown. The recorder is provided with a notebook in which to enter depths of soundings, nature of bottom, etc.

13. Making the Soundings.—If the depth of the water does not exceed about 75 feet, the soundings can usually be made while the boat is in motion. When the soundings are made at long intervals and the depth of the water does not exceed about 30 feet, it is usually more advantageous to withdraw the lead from the water after each sounding. In this case the lead is cast far enough ahead of the boat, as each sounding is made, for the line to become vertical when the lead reaches the bottom. If the depth of the water is too great for this method, the soundings can be made at intervals, as the boat moves, without drawing up the lead farther between soundings than is necessary to free it from the bottom.

As the soundings are made, the leadsman calls out the observed depth of each sounding to the recorder, who repeats the depth to prevent mistakes and then enters it in his notebook, together with the time and the number of the sounding. The character of the bottom is observed and noted at such intervals as may be desired, and all changes in the material of the bottom are noted.

GAUGES

14. Tide Gauges.—The bottom depths, as determined by the soundings, are measured from the surface of the water, the elevation of which varies considerably in river and tidal waters. In order to reduce the observed depths to the same surface of reference or to the datum of the survey, it is necessary to know the water level at the time each sounding is made. For this purpose a gauge that will show the height of the water surface should be established at some convenient place. An ordinary graduated board or staff is best for temporary use. This may consist of a board about 6 inches wide, 1 inch thick, and of a length somewhat greater than the variation in the height of the water, painted white and

graduated to feet and tenths in black. Such gauges are used very commonly for this purpose, and are called **staff gauges.**

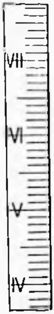


Fig. 7

A simple form of staff gauge is shown in Fig. 7. For facilitating the reading of the gauge, a float, consisting of a small board painted white, may be so placed as to rest on the water surface in front of the gauge. This float moves up or down with the water surface as it rises or falls, and indicates at once the gauge reading.

15. Location of Gauges.—A tide gauge may be attached to a dock, quay wall, pile, stake, tree, or any other stationary object that is in a convenient position and to which the gauge can be secured in a vertical position. Sometimes the gauge is divided into sections and fastened to different objects, as trees, according to their heights, as illustrated in Fig. 8. Each section should slightly overlap the other, so as to afford a continuous gauge reading. In some cases, the gauge may be conveniently set at any suitable inclination and attached to stakes driven firmly in the bank. It may be made in sections and fastened to stakes in such a manner as to conform to the slope of the bank, as illustrated in Fig. 9. Each section should consist of a straight, well-seasoned board from 4 to 6 inches wide and 1 inch thick, and should be painted white. The divisions are determined with the level and should be marked by nails or tacks driven into the face of the board. Such a gauge should be located where the bank is not changing by caving off or filling up, and should be in

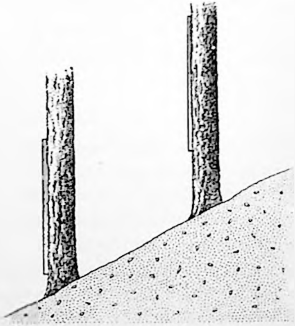


Fig. 8

a location where it will not be disturbed by floating drift at periods of high water. A gauge so placed can be easily observed from the bank at any height of the water.

When a continuous record of the fluctuations of the water surface in tidal waters is desired, a self-registering gauge should be used. This consists essentially of a float that rises and falls with the tide. The float is protected by a perforated box and is so arranged that its motion is recorded by a stylus or pencil on a roll of paper, which passes over a cylinder that is revolved at a uniform speed by clockwork. The path of the pencil on the paper indicates the stage of the water at any given time.

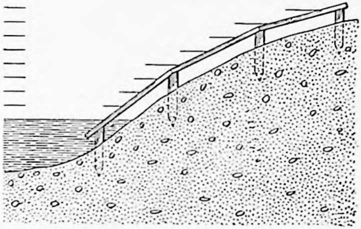


FIG. 9

THE SEXTANT

16. Law of Reflection.—Let AB, Fig. 10, be a plane mirror; PO, a ray of light meeting the mirror at O; OP', the direction taken by the ray after it strikes the mirror; and ON, a line perpendicular to the plane of the mirror. This perpendicular is called the **normal** to the mirror at O. The ray PO coming to the mirror is called the **incident ray**, and the angle NOP that it makes with the normal is called the **angle of incidence.** The ray OP' leaving the mirror is called the **reflected ray**, and the angle NOP' that it makes with the normal is called the **angle of reflection.** It is a general law of physics that the angle of incidence is always equal to the angle of reflection; that is, $NOP = NOP'$. It follows that the angles POA and $P'OB$ are also equal.

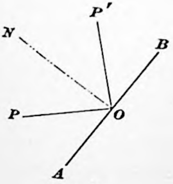


FIG. 10

17. Description of Sextant.—The sextant is a hand instrument for measuring angles. By means of it the angle between two lines of sight can be measured by a single operation. The angle between two lines of sight directed to two objects is commonly spoken of as the **angular distance** between the objects. When making the observations, the instrument is held in the hand, and successive angular measurements can be made with great rapidity. It is therefore especially adapted for use in a boat on the water, where the motion renders the use of fixed instruments impracticable.

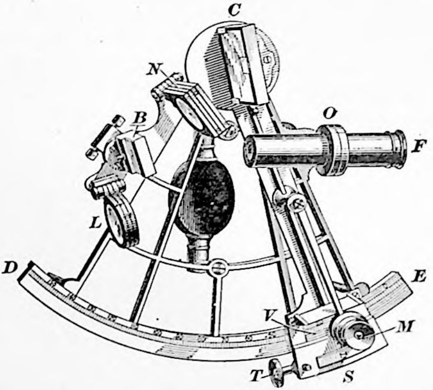


FIG. 11

As the sextant is frequently used in the location of soundings, a description of the instrument and a discussion of its theory are given here.

A sextant, as represented in Fig. 11, consists of a metal frame CDE, and an arm CS, called the **index arm**, which is fitted with a vernier and rotates about the center C of the sextant; to this index arm is also attached the **index mirror** BC, Fig. 12. To the arm CD is fixed the **horizon glass** A, half of the back of which is silvered, while the other half is transparent. The arm CE carries a **telescope** T directed

toward the horizon glass A. Thus, while the telescope is directed to an object H, the rays of light from another body S are reflected first from the mirror BC to the silvered half of the mirror A, and then from this mirror to the telescope in the direction HAT. The observer will thus see both of the bodies H and S in the field of the telescope together.

In order that the ray of light SC may enter the telescope after reflection, the index arm BI must be turned about the pivot C until the mirror BC is brought into the proper position with reference to SC and A. When I is at E, the two mirrors are parallel; and when I has been moved forwards until the rays of light SC, after two reflections, enter the telescope, the arc EI over which the arm BI carrying the mirror BC has been moved will be exactly equal to one-half the angle between SC and HT. Thus, the angle between the rays of light coming from two distant objects H and S may be measured as follows: The observer points the telescope directly toward H and then moves the arm I along the arc ED until the reflected ray $SCAT$ also passes along the line HAT and the image of the second object enters the telescope. The arc EI, which is equal to the angle, called the **index angle**, between the surfaces of the index mirror CB and the horizon glass A, will then be one-half the angle between SC and HA, that is, one-half the apparent angular distance of the bodies H and S.

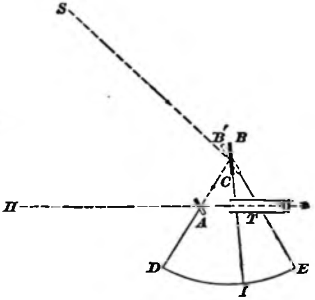


FIG. 12

Since the angle ECI is one-half the true angle between SC and HA, the arc ED has each half degree marked as a whole degree, so that ED, which is an arc of 60°, is divided into 120 equal parts and each part marked 1°. This is done merely to spare the observer the trouble of multiplying the reading by 2.

18. Theory of the Sextant.—That the index angle, or angle between the index mirror BC and the horizon glass A is equal to one-half the angle between SC and HA, Fig. 13, will now be proved. With the index arm in any position, produce the planes of the mirrors to their intersection at O, and produce the perpendiculars to the mirrors, pC and $p'A$, to their intersection at N. Since CE is the zero position of the index arm and CO its given position, the angle ECO is the index angle. Since CE is parallel to AO, $ECO = COA$, and since pC and $p'A$ are perpendicular to CO and AO, respectively, $COA = pNA$. Hence, $pNA = N$ is equal to the index angle.

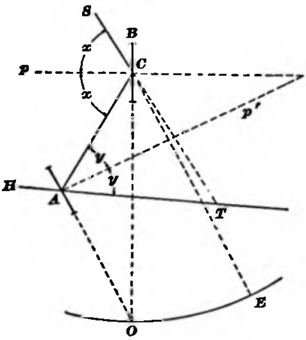


Fig. 13

It will be remembered that the ray of light SC, before reflection, makes the same angle with the perpendicular Cp as the ray of light CA after reflection. Hence, $SCp = pCA$, and similarly $CAp' = p'AT$. Calling each of the former angles x, for brevity, and each of the latter angles y, we have from the triangle ACN,

$$N = x - y$$

The observed angle is $STH = T$. From the triangle ACT,

$$T = 2x - 2y = 2(x - y) = 2N$$

whence, $$N = \tfrac{1}{2}T$$

That is, the angle between the two mirrors, or the index angle, is equal to one-half the angular distance between the two objects.

The vernier plate V, Fig. 11, is attached to the index bar immediately below the graduations on the limb. The index bar can be clamped to the limb by means of a clamp screw (not shown in the figure), and when so clamped can be

moved very slowly by means of the tangent screw T. The magnifying glass M is for reading the graduations on the limb and vernier. The two sets of colored-glass shades N and L are used when the sun is observed, in order to prevent the glare of its light from affecting the observer's eye; they are attached to the frame by hinges in such manner that the shades N can be turned into the path of the reflected ray and the shades L into the path of the direct ray.

19. The Vernier.—As stated previously, the divisions on the limb of a sextant correspond to degrees, and each division is subdivided into three, four, or six parts, according to the instrument. These divisions are numbered as shown in Fig. 14, the upper portion of which represents part of the limb of a sextant. The divisions representing degrees on the limb there shown are subdivided into three parts, each part representing a third of a degree, or 20 minutes. By means of the vernier shown

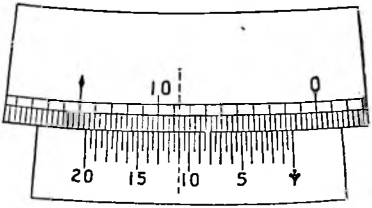


FIG. 14

in the lower part of the figure, however, the angles can be read to half minutes, or 30 seconds. The vernier of a sextant is substantially the same as that of a transit, except that it is single instead of double; that is, it reads in only one direction, to the left, from the zero point. instead of both to the left and to the right. The first line at the extreme right of the sextant vernier, which is usually designated by a spear-shaped mark, is the zero point or index mark of the vernier. The vernier shown in Fig. 14 is divided into forty equal parts, which together are equal to thirty-nine divisions on the limb. Since the least reading is equal to a scale division divided by the number of parts in the vernier, it is $\frac{20}{40}$ minute, or 30 seconds. The numbering of the vernier corresponds to whole

minutes, which are represented by the longer graduations. the shorter graduations represent half minutes. Some sextants are graduated to read to 20 seconds and some to 10 seconds.

The graduations of the limb of a sextant usually extend a few degrees to the right of the zero mark, and it is sometimes convenient to make an observation that requires the vernier to be read when the index mark stands to the right of the zero mark. Such readings are said to be *off the arc.* When readings are made off the arc, the graduations are counted and the vernier is read to the right instead of to the left. The number of degrees and minutes are counted to the right from the zero mark to the first graduation at the left of the index mark; to this is added the number of minutes indicated by the coincident line of the vernier, as counted to the right from the graduation at the extreme left of the vernier. Thus, in Fig. 15, the index mark of the vernier stands to the right of the zero mark of the limb and the reading is off the arc. The angle is read by counting to the right on both the limb and vernier, as just described.

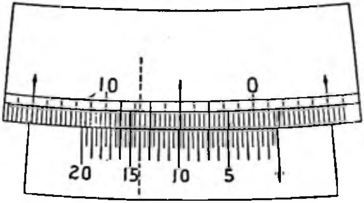


FIG. 15

20. Adjustments of the Sextant.—There are four adjustments of the sextant, as follows:

1. To make the plane of the index glass perpendicular to the plane of the limb.

2. To make the plane of the horizon glass perpendicular to the plane of the limb.

3. To make the line of collimation of the telescope parallel to the plane of the limb.

4. To make the planes of the mirrors parallel when the index reading is zero.

21. To Adjust the Index Glass.—Place the index bar near the middle of the limb; with the eye near the plane of the limb, observe whether the limb as seen directly and its image as reflected in the index glass form a smooth continuous curve; if they do, the glass is perpendicular to the plane of the limb and the adjustment is correct. But if the reflected limb appears to be above that part of the limb seen directly, the glass leans forward; if the reflected limb appears to be below, the glass leans backward. In either case, the glass is made perpendicular to the plane of the limb by means of the adjusting screws at its base.

22. To Adjust the Horizon Glass.—Look through the telescope and horizon glass toward a star or other well-defined distant object. Move the index bar slowly until the reflected image passes over the image seen directly. If these images coincide, the horizon glass is perpendicular to the plane of the limb. If they do not coincide, the horizon glass is adjusted by an adjusting screw placed under, behind, or beside the glass, according to the construction of the instrument.

23. To Adjust the Telescope.—Place the sextant in a horizontal position on a table or other support, and direct the telescope at some well-defined point or mark about 20 feet away. Place two small blocks of equal height on the limb, one near each extremity. These blocks should be of exactly equal height, so that a line of sight over their tops will be parallel to the plane of the limb, and should be at the same height above the limb as the center of the telescope. Some sextants are provided with two small brass sights that can be placed on the limb for this purpose. Sight over the tops of the two blocks or through the sights, as the case may be, in the direction of the point or mark sighted through the telescope, and note if the line of sight intersects the mark. If it does not, but falls above or below the mark, the telescope is not parallel to the limb. It can be made parallel to the limb by means of the screws in the collar that holds the telescope. This adjustment, however, is not usually made

unless the error is considerable, since a slight lack of parallelism between the line of sight and the plane of the limb does not appreciably affect the angular measurements on the limb.

24. To Adjust Index or Find Index Error.—Set the index at zero, look through the telescope toward a star and note whether the direct and reflected images of the star coincide. If they do, the adjustment is correct. If they do not, move the index bar until they do coincide, and clamp it in this position. The reading of the index when in this position is called the **index error.** This error can be corrected by means of screws at the back of the index glass, which cause it to revolve about an axis perpendicular to the plane of the limb. To make the correction, set the index bar at zero and, by turning the screws, revolve the index glass until the two images exactly coincide. This adjustment will usually disturb the previous adjustment of the index glass, and, as a rule, it is not made unless the index error is greater than 3 minutes.

When the index error is less than 3 minutes, it is usually applied as a correction to all observations. If the error is *off* the arc, that is, if the index is to the right of the zero mark, it is additive or plus and should be added to all readings. If the error is *on* the arc, that is, if the index is to the left of the zero mark, the error is subtractive or minus and should be subtracted from all readings.

25. Method of Using the Sextant.—To measure an angle between two objects with a sextant, hold the plane of the limb in the plane of the two objects, look through the telescope toward the less distinct object, and move the index bar until the reflected image of the brighter object comes in contact with the direct image of the less distinct object. Clamp the index bar, and, with the tangent screw, bring the two images exactly together. Note the reading of the vernier and apply the correction for the index error.

In order to have the plane of the limb in the plane of the two objects when the telescope is directed toward the less

distinct object, it may sometimes be necessary to hold the sextant upside down. In locating soundings, the measured angles should lie in planes that are almost horizontal.

EXAMPLE.—The angular distance between two objects, as measured with a sextant, reads on the vernier 35° 36′ 30″; what is the true angular distance if the index error of the sextant is: (*a*) +1′ 20″: (*b*) −1′ 40″?

SOLUTION.—(*a*) Since the vernier reading is 35° 36′ 30″ and the index error is +1′ 20″, the true angular distance is equal to 35° 36′ 30″ + 1′ 20″ = 35° 37′ 50″. Ans.

(*b*) Since in this case the index error is −1′ 40″, the true angular distance is equal to 35° 36′ 30″ − 1′ 40″ = 35° 34′ 50″. Ans.

LOCATING SOUNDINGS

RANGES

26. Preliminary Remarks.—Before starting the sounding work, the stations, triangulation points, and ranges should be carefully located. The work should be so arranged that the soundings may be made and located as rapidly as possible, especially when the area to be sounded is large, or many soundings are to be made. The position of the sun should be considered, so that clear, distinct sights may be had without interference from glare. The observer should be so stationed that while making observations the sun is not directly in his face, but preferably on his back or overhead. If practicable, the order of work should be so arranged that observations toward the west may be taken in the forenoon, and toward the east in the afternoon. In tidal waters, the range of the tide should be considered. If the difference in elevation between high and low tide is very great, sounding work should preferably begin after the tide has fallen to a level about half way between high and low tide, or after half ebb, and cease when it has risen to the same level or at half flood. Usually, however, sounding work can be done at all stages of the tide.

In all sounding operations where simultaneous measurements are to be made, the recorder and the various observers

should have watches set accurately to the same time. Soundings are usually made at regular intervals of time, but there is no fixed rule regarding this. The length of time between successive soundings will depend on the depth of the water, the method of observation, and the distance between adjacent soundings. When the soundings are located from the shore with a transit, the observations are commonly made at intervals of 1 minute. In this case, intermediate soundings may be located by interpolation. When soundings are located from a sounding boat by sextant observations, as many as three observations per minute can be made, since angles can be measured more rapidly with a sextant than with a transit.

27. Sounding Ranges.—Soundings are usually made on well-defined lines or courses whose positions are known. These lines are called **ranges.** They are usually laid out on shore and prolonged across the area of water surface to be sounded. In such cases, two points on each range are selected at which poles or other signals are placed to serve as guides to the sounding party in determining the range. These points should be a considerable distance apart, and should be carefully located in order to accurately establish the direction of the range passing through them. The point near the shore is called the **front range point,** or **front signal,** and that back from the shore is called the **back range point,** or **back signal.** The steersman determines the course of the sounding boat by sighting along the range and keeping the two range points in line.

The ranges should be laid out and arranged with reference to the extent of the area to be sounded and also the method of locating the soundings. In localities where the area to be sounded is comparatively small, the arrangement illustrated in Fig. 16 can be used. Two parallel rows of stakes, as *C D* and *E F*, are established. The stakes in each row through which the parallel ranges pass are usually spaced at regular intervals. The length of the interval or the distance between adjacent ranges will depend on the frequency with which the soundings are to be made. The distance between the front

and back range points should be such that well-defined ranges can be established by sighting from the boat to two range points in line.

In Fig. 16, the dotted parallel lines represent the sounding ranges. A and B are observation stations so situated as to offer a clear view over the area to be sounded, and preferably visible one from the other; the distance between them should be accurately determined.

This system of range lines controls the positions of the soundings, which are made on the range line and distributed over the area to be sounded in such manner as may be desired, thus avoiding unequal distribution, making two or more soundings at or near the same place, or allowing too great an interval between soundings. For such purpose, the range lines are advantageous in nearly all soundings. When a boat is on a range the observer always knows its approximate position. Soundings made on range lines are platted with greater facility than those not made on ranges. The system of range lines is especially adapted for use when the soundings are located from shore by means of angles measured with a transit at one extremity of the base line. One angle for each sounding is measured between the base line and the line of sight to the boat. Before commencing the soundings, the angles made by the range lines with the base line should be measured, and the distance along the base line from the instrument to the intersection of each range line with the base should be determined. In some cases, the soundings are located by means of two angles measured simultaneously from both extremities of the base to the sounding boat; in this case each range is used as a guide for the sounding boat and also as a check on the accuracy of the locations made on it.

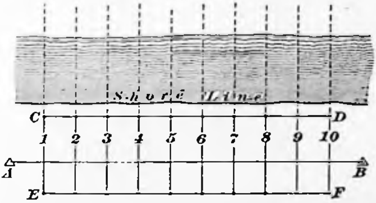


FIG. 16

28. Ranges Marked by Buoys.—If the shore is heavily wooded or rocky and precipitous, it may be impracticable to establish two rows of stakes at a sufficient distance apart to serve as front and back range points. In such a case, buoys may be used for signals to mark the front range points. Each range will then consist of a point on shore and a buoy in the water, as illustrated in Fig. 17. In this figure, *A*, *B*, *C*, and *D* are points or stations on shore, which may be located by direct measurement from point to point, while *1*, *2*, *3*, and *4* are the corresponding buoys. The buoys are located by measuring two angles in each triangle, the angles being read from the shore stations. Thus, the buoy at *1* can be located by measuring the angles *B A 1* and *A B 1*; the buoy

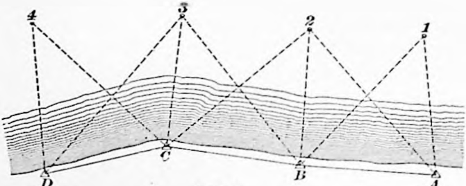


Fig. 17

at 2 by measuring the angles *A B 2* and *B A 2*, or the angles *C B 2* and *B C 2*. The buoys at *3* and *4* are located in a similar manner.

Buoys may be used for points of reference in connection with fixed points on shore. The position of each buoy will vary within small limits according to the stage of the tide, but such variations will not usually be sufficient to cause appreciable error in the platted positions of the soundings.

29. Radial Ranges.—Where topographical conditions will permit, the front range points are located close to the shore line, and some prominent natural object, such as a tall tree, a church spire, a windmill, or the cupola of a building, is selected for a back range signal. In such a case, the distance from the shore line to the back range signal should be such that radial range lines from this point

through the several front range signals will cover the area to be sounded. The distance between adjacent ranges at their extremities should not usually be greater than the average distance between successive soundings on a range. Such an arrangement is illustrated in Fig. 18. In this figure AB is a base line on which the front range points are located; these are represented by small dots. The broken lines numbered *1*, *2*, *3*, etc. are the sounding ranges. The front range points should be at known distances apart, the distance between adjacent range points being carefully measured with a steel tape, and each point marked by a stake. The points A and B at the extremities of the base line, and

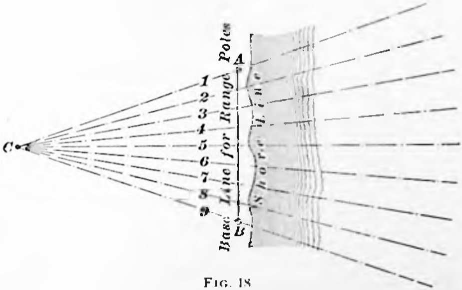


FIG. 18

the back range signal C, can be located from other points on the outline survey whose positions are known.

30. Ranges Across Streams.—When soundings are to be made across rivers or streams of considerable magnitude, sounding ranges are usually run across in directions perpendicular to the axis of the stream. Such ranges are marked by range signals placed either on one or both banks according to the width of the stream.

RANGE SIGNALS

31. Range Poles.—In designating the ranges, it is important to have the front and back range points marked by poles, or other objects, sufficiently conspicuous to be

easily visible from the sounding boat, and designated in such a manner as to distinguish the different ranges, each from the others. Such objects are called **signals**; they may consist of poles or rods, or of natural objects on the shore. When a range point is in the water, the signal marking it usually consists of a buoy, as has been previously stated. In shallow water, signals similar to those used on the shore are often used, being set on, or driven into, the bottom.

Range signals on shore usually consist of poles of suitable height and dimensions. When the sounding ranges are short, ordinary transit sight poles are frequently used for signals. In such cases an assistant holds the rod in position on the stake marking the range point, or it is placed in a vertical position by the stake.

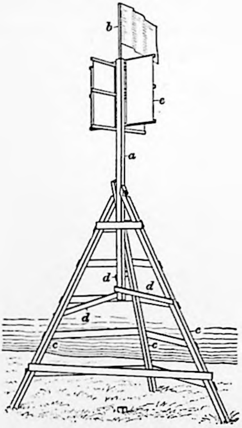


FIG. 19

If the ranges are of considerable length and soundings are to be made at some distance from shore, larger and more conspicuous poles should be used for signals. These may consist of pieces of 4″ × 4″ scantling, of suitable lengths. They should be set vertically and may be supported by being firmly driven into the ground, their lower ends being sharpened, or by being placed in holes dug for the purpose. If the ground is hard or rocky, the poles may be supported by braces or by stones piled around their bases. A form of range signal that is often used is illustrated in Fig. 19. A pole *a*, consisting of a piece of 4″×4″ scantling of suitable length and carrying a flagstaff *b* on top of it, is set in a vertical position with its bottom at any convenient height above the ground. The pole is held in position by three inclined braces, or legs *c*, which are bolted to it, and by horizontal wooden strips *d*

nailed to the pole and the legs. Other horizontal strips are attached to the legs to keep them from spreading. To make the signal more conspicuous, a crossed target *e* of wood and cloth may be nailed to the pole *a* near its upper end. This form of range signal can be set directly over a stake or hub, as shown in the illustration.

FIG. 20

Range poles should be whitewashed so as to be conspicuous against the background of the shore. When a number of adjacent ranges are used, they may be designated by attaching colored flags to the poles or signals. In such cases each range is known by some distinctive color or combination of colors. Thus, the flag for range No. *1* may be red; that for range No. *2*, red and white, etc.

Ranges are sometimes designated by strips of wood, such as laths or barrel staves, nailed to the poles or signals. The strips should be arranged in the form of Roman numerals, as shown in Fig. 20. In such cases the numerals denote the numbers of the ranges; the arrangement shown in Fig. 20 represents range No. *6*. The strips and the pole should be whitewashed so as to be conspicuous.

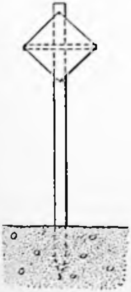


FIG. 21

32. Targets.—When sounding ranges are to be projected a considerable distance from shore, the range poles or signals should be provided with targets in order to be conspicuous. Such targets should be sufficiently large to be visible from the sounding boat at the most distant point on the range, and of suitable designs or colors to enable the steersman to distinguish readily the different ranges. No rigid rule can be laid down specifying any particular form or design for range targets. They may be made of such forms as are best suited for the purpose, and composed of such materials as are available in each case.

A good form of target for range signals is shown in Fig. 21. The target is lozenge shaped and can be made in the following manner: A strip of wood, about 3 inches wide, 1 inch thick, and 3 feet long, is nailed to the face of the range pole about 3 feet below the top, care being taken to center the strip and to make it perpendicular to the pole. To the framework thus formed, a square piece of white or colored cloth is tacked in such a manner that two diagonally opposite corners of the cloth are at the two extremities of the cross-piece, the two other corners being on the center line of the range pole. Such a target is quickly and economically made and is very effective when the sights are directly in front and the full size of the target is visible.

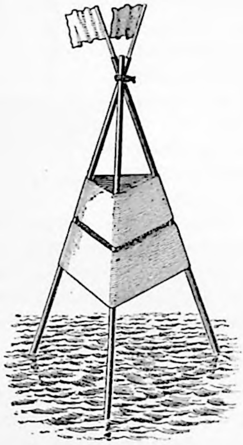


FIG. 22

33. Range Signals in the Water.—When a range point is in the water, the signal marking it usually consists of a buoy; in shallow water, however, stationary range signals are often used. A form of signal used by the United States Coast and Geodetic Survey is shown in Fig. 22. This consists of a tripod about 10 or 12 feet high, each leg being made of a piece of gas pipe about $1\frac{1}{2}$ inches in diameter. The legs are forced firmly into the mud or sand of the bottom, at suitable distances apart, and inclined toward the center. They are lashed together near the top, and flags about $1\frac{1}{2}$ feet square attached to poles just large enough to fit inside the pipes are placed at their upper ends as shown. Two strips of cloth, each about $\frac{1}{2}$ yard wide, are wrapped around the tripod, about half way between its top and bottom. These strips serve as a target.

34. Buoys.—A buoy is a float of wood or other suitable material, or a hollow air-tight vessel, anchored in place by a heavy weight to which it is attached by a rope or chain. Buoys are used to mark certain places or points on the water surface. They are usually employed to designate the limits of a channel or some submerged object in connection with navigation. They are also used as points of reference and for range points in hydrographic surveying, and only such as are suitable for such purposes will be considered here.

A form of buoy that has been much used in hydrographic surveying is illustrated in Fig. 23. It consists essentially of a round log of cedar or other light wood, about 1 foot in diameter and 3 feet in length, sawed square at ends. The lower half is trimmed in the shape of a truncated cone, tapering to about 5 inches in diameter at the lower end. A hole about 2 inches in diameter and 9 inches deep is bored into the lower end on the axis of the log, into which a pole 2 inches in diameter is driven. The upper end of the pole is split, and a wedge inserted in the cleft, which is driven up and tightens the pole as it is driven to the end of the socket, thus preventing the pole from pulling out. A hole sufficiently large for the anchor rope to pass through is bored through the pole a few inches above its lower end. The anchor rope is preferably of manila hemp and is about 1 inch in diameter. A good form of knot with which to tie the rope is shown in the figure; it is shown open in order to illustrate the method of forming the knot. This knot is called the **two half-hitches.** A hole about 2 inches in diameter and 10 inches deep is bored into the top of the buoy along its axis. This serves as a socket into which is inserted a staff

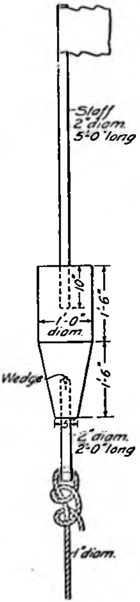


FIG. 23

about 2 inches in diameter and 5 feet long. At the upper end of this staff is fastened a flag about 1 foot square and of suitable design or color.

Another form of buoy is illustrated in Fig. 24. This consists of a log or round piece of light wood, about 3 feet long, sawed square at both ends. This log is trimmed in the shape of a truncated cone, tapering from about 8 or 10 inches in diameter at the upper end to a diameter of about 4 or 5 inches at the lower end. A hole about 2 inches in diameter is bored completely through the log, the center of the hole coinciding with the axis of the log, and a round pole large enough to fit the hole closely is passed through the log and wedged tightly in place. This pole should project about 2 or 3 feet below the bottom of the log or buoy and about 3 or 4 feet above its top. A flag about 1 foot square is fastened to the upper end of the pole as shown. The anchor rope is passed through a hole in the pole near its lower end and tied as described in the preceding article.

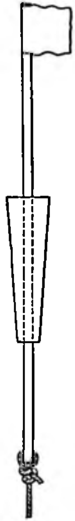


Fig. 24

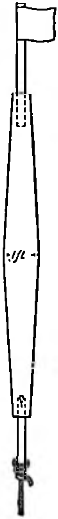


Fig. 25

35. Buoys for Tidal Waters.—The two forms of buoys just described are suitable for use in non-tidal waters and in rivers and streams where the current is not sufficiently strong to drag the top of the buoy under or level with the surface of the water.

Buoys for use in tidal waters should be of sufficient length to be visible at all stages of the tide. The best length for a buoy in a given tidal water will depend on the range of the tide, and is usually greater than that required for buoys in non-tidal waters.

A good form of buoy for use in tidal waters is illustrated in Fig. 25. The length shown is 10 feet, but this may be varied to conform to the range of the tide. The buoy is

made from a log or round piece of light wood and trimmed so as to taper gradually from the middle to each end. It has a diameter of about 1 foot at the middle and tapers to a diameter of from 4 to 6 inches at each end. It is sawed square at the ends, and holes are bored at each end, into which poles are inserted for attaching the flag and anchor rope, as previously described. If preferred, a ring may be fastened to the lower end of the buoy by means of a staple or screw, and used for securing the anchor rope to the buoy.

A more simple form of buoy, consisting of a single stick or log of timber of suitable length and diameter, is often used in tidal waters; the shape is similar to that illustrated in Fig. 25, but the buoy is more uniform in size from top to bottom. Such buoys are commonly called **spar buoys.**

METHODS OF LOCATING SOUNDINGS

36. Methods Employed.—In order to plat soundings correctly on a map or chart, the position of each sounding must be located; that is, its relative position with respect to known points on shore must be determined. Soundings can be located by various methods, depending on local conditions, the object of the survey, and the degree of accuracy required. The following list comprises the best known and most frequently used methods of locating soundings: (1) by time intervals; (2) by one angle measured on shore; (3) by two angles measured simultaneously on shore; (4) by two angles measured in the sounding boat; (5) by transit and stadia; (6) by a fixed line marked by a wire or rope; (7) by the intersection of fixed ranges. These methods will be described in order.

37. By Time Intervals.—When this method is employed, the soundings are made at stated intervals of time while the sounding boat moves at uniform speed along a range or on a course not marked by range signals. The soundings may be made under two conditions, namely: (*a*) The first and last soundings on a range or course are located by observation and all the intermediate soundings are

located by interpolation or time intervals. (*b*) Alternate soundings or those made at convenient intervals are located by observation, and such intermediate soundings as are not observed are located by interpolation. In either case the method of interpolating the intermediate soundings is the same. Knowing the distance between the two end soundings or between two adjacent observed soundings, the time interval between them, and the time interval between consecutive soundings, the position of each intermediate sounding can be determined by proportion as follows:

Let T = time elapsed between two observed soundings;
D = distance between these observed soundings;
t = time interval between consecutive soundings;
d = distance between consecutive soundings.

Then, if the boat moves at a uniform speed,

$$D : T = d : t$$

from which

$$d = \frac{D\,t}{T}$$

However, since the speed of the boat is likely to vary, this method of locating soundings is not very accurate.

EXAMPLE.—A sounding boat moving at a uniform speed traverses a range 1,800 feet long in 20 minutes, and a sounding is made at each end of the range and at intervals of 1 minute; what is the distance between consecutive soundings?

SOLUTION.—The two end soundings are 1,800 ft. apart = D; the elapsed time between them is 20 min. = T; and the time interval t between consecutive soundings is 1 min. Substituting these values in the formula gives,

$$d = \frac{1{,}800 \times 1}{20} = 90 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. A range 500 feet long is traversed at uniform speed in 10 minutes by a sounding boat from which soundings are made at intervals of $\frac{1}{2}$ minute; find the distance between any two consecutive soundings.

Ans. 25 ft.

2. In the preceding example, if the soundings are numbered consecutively 1, 2, 3, etc., from beginning to end of the range, what is the distance between soundings Nos. 5 and 12?

Ans. 175 ft.

3. Soundings are made at intervals of 15 seconds or at the rate of 4 per minute, from a boat moving along a course at a uniform speed; the soundings made at the end of each minute are located by observations and the intermediate soundings are interpolated. The observed soundings, when located, are found to be at intervals of 204 feet apart; what is the distance between consecutive soundings? Ans. 51 ft.

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38. By One Angle Measured on Shore.—When this method is used, the boat containing the sounding party traverses a fixed range while an observer on shore measures the angle between a base line and the line of sight to the leadsman at the time a given sounding is made. The ranges are usually parallel to each other and are preferably at right angles to the base line. They should be at known distances apart and the distance from the observation station to each range should be determined by careful measurement along the base line. The base line may have an observation station at each extremity, in which case the observer is stationed at either end, as may be most convenient, and orients his instrument by sighting toward the station at the other end. In Fig. 26 is shown a base line at the ends of which are two observation stations A and B, whose positions have been determined. The ranges numbered 1, 2, 3, etc. are parallel to each other and at right angles to the base line. Each range is designated by front and back range signals whose positions are shown by the small dots in the figure. The

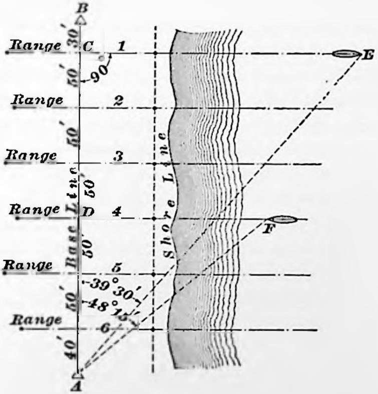

FIG. 26

distances along the base line between the observation stations and the nearest ranges and the distances between adjacent ranges are as shown by the figures.

The field party usually consists of the observer on shore, and in the boat the recorder, the leadsman, the signalman, and the boat crew. In tidal waters, a tide-gauge reader should be added to the shore party; his duties consist in reading the tide gauge every 5 minutes during sounding operations and recording the times and gauge readings. The watches of the observer on shore and the recorder in the boat are set accurately to the same time. The sounding boat traverses the ranges in succession and soundings are made at regular intervals of time, which depend on the depth of the water and on the accuracy required. Usually, from one to four soundings are made per minute. A few seconds before the end of each interval the recorder has the signalman raise his flag, and exactly at the end of the interval the flag is lowered, as a signal to the observer that the sounding has been made. The leadsman calls out the observed depth of each sounding to the recorder, who enters it in his notebook together with the number of the sounding. The exact time of each sounding is recorded. The character of the bottom is observed by the leadsman and noted by the recorder at such intervals as may be required.

The observer is stationed at A with his transit set up over the transit point at that end of the base line. The vernier of the transit is first set at zero and the telescope is directed toward B, the point at the other end of the base line, and the instrument is clamped. The upper plate is then unclamped and the observer turns the instrument in azimuth toward the sounding boat. When making the observations, the observer usually keeps his watch open and lying face uppermost on the upper plate of the transit, for convenience in noting the time. When the signal flag is raised in the boat the observer sights through the telescope toward it, and by turning the upper plate slowly and carefully, keeps the line of sight fixed on the flag. At the instant the flag is lowered the observer ceases to turn the plate. He notes the exact

time and reads the angle on the plate and enters the number of the sounding, the time, and the angle in his notebook. The observations are usually made on the flag, as it can be seen more distinctly and at greater distances than a sounding line or pole. On this account the signalman should be stationed near the leadsman, so that the difference between the observed and the true position of each sounding will be small. The upper plate of the transit should remain unclamped while observations are being made on the sounding boat, since there is usually not sufficient time between observations to permit its being clamped. Also, frequent clamping and unclamping would tend to disturb the position of the transit and thus may introduce errors in the observed angles.

In some instances, there is at each transit station a recorder in addition to the observer, and it is the duty of the recorder to enter in the notebook the observations called out to him by the observer. Thus, he records the number of each sounding, the time it was made, and the observed angle. Furthermore, it is customary for the signalman to raise a red flag for every fifth sounding and a white flag for the intermediate soundings. Whenever the observer sees the red flag displayed by the signalman he calls out "red" to the recorder, who then notes the number of the sounding, which should be a multiple of five, in order to correspond with the number entered by the recorder in the boat. In this way a check is obtained on the numbering of the soundings, and any difference in the numbering by the recorder in the sounding boat and the recorder on shore can be readily detected.

EXAMPLE.—The ranges shown in Fig. 26 are at right angles to the base line; a sounding is made while the sounding boat is at E on range *1*. The observed angle EAB is 39° 30′ and the distance AC along the base line from A to the intersection C of range *1* is 290 feet; find: (*a*) the distance AE; (*b*) the distance along the range line from C, its intersection with the base line to the sounding at E.

SOLUTION.—(*a*) Since the ranges are at right angles to the base line, AE is the hypotenuse of the right triangle ACE. From trigonometry,

$$AE = \frac{AC}{\cos EAC} = \frac{290}{\cos 39° 30'} = 375.8 \text{ ft.} \quad \text{Ans.}$$

(*b*) The distance from E along range *1* to its intersection C with the base line is $EC = AC \tan EAC = 290 \tan 39° 30' = 239.1$ ft., nearly. Ans.

EXAMPLES FOR PRACTICE

1. The distance AD along the base line from A to the intersection of range *4*, Fig. 26, is 140 feet; the angle FAB is 48° 15′; find: (*a*) the distance AF; (*b*) the distance along the range from F to its intersection D with the base line. Ans. $\begin{cases}(a) & 210.2 \text{ ft.}\\(b) & 156.9 \text{ ft.}\end{cases}$

2. A sounding boat is on a range perpendicular to the base line. The angle measured at an observation station between the base line and the line of sight to the boat at the time a given sounding is made is 30° 30′; the distance from the station to the intersection of the base line and range is 230 feet; find: (*a*) the distance from the station to the position of the sounding; (*b*) the distance along the range line from the base line to the position of the sounding. Ans. $\begin{cases}(a) & 266.9 \text{ ft.}\\(b) & 135.5 \text{ ft.}\end{cases}$

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39. By Two Angles Measured Simultaneously on Shore.—This is one of the commonest methods, and if the work is carefully done it is both convenient and accurate. Two observers are required, each occupying a station whose position with respect to the shore survey has been determined. The observation stations should be so situated as to afford a clear field of view over the area to be surveyed, and when possible should be visible one from another. They may be at the extremities of a base line, whose length has been carefully measured, or at two points whose positions and distance apart have been determined by triangulation. The vernier of each instrument is set to read zero when the telescope is directed toward the other instrument point or toward some common point whose position is known. The field work is entirely similar to that described for the preceding method, and the field party and equipment is the same with the addition of an observer and transit. This method differs from the preceding method, however, in that two angles, instead of one, are measured for each location, the observations being made simultaneously, and the position of the observed sounding is determined by the intersection of the two lines of sight from the two observation stations.

instead of by the intersection of one line of sight with a range line. In using this method it is not necessary to have the sounding ranges parallel or to lay them out at right angles to the base line, although such an arrangement is advantageous in affording a means of checking the accuracy of the angular measurement. For locating soundings over a limited area by this method, the arrangement shown in Fig. 27 is convenient and gives good results. In this figure, A and B are the observation stations at the extremities of a base line AB. The ranges are shown by the parallel lines passing through the small dots indicating the range signals. One position of the sounding boat is shown at C, and the lines of sight to this position from the two observation stations are represented by the dotted lines AC and BC.

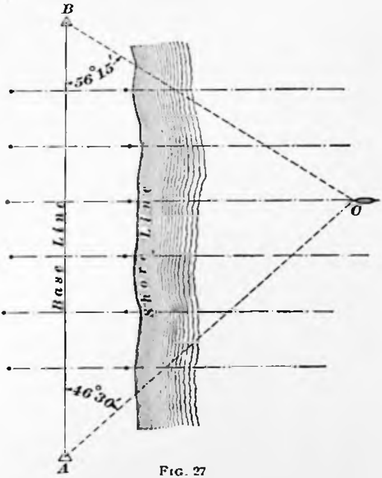

FIG. 27

EXAMPLE.—A sounding is made while the sounding boat is at the position C, Fig. 27. The angles observed from A and B are 46° 30′ and 56° 15′, respectively, as shown, and the length of the base line AB is 785 feet; find the distances AC and BC, giving values to the nearest foot.

SOLUTION.—The angle $BAC = 46° 30'$, and $ABC = 56° 15'$; hence, $ACB = 180° - (46° 30' + 56° 15') = 77° 15'$, and, from trigonometry,

$$AC = \frac{AB \sin ABC}{\sin ACB} = \frac{785 \sin 56° 15'}{\sin 77° 15'} = 669 \text{ ft. Ans.}$$

$$BC = \frac{AB \sin BAC}{\sin ACB} = \frac{785 \sin 46° 30'}{\sin 77° 15'} = 584 \text{ ft. Ans.}$$

40. By Two Angles Measured in the Sounding Boat.—This method, which is used extensively in harbor work, is one of the best general methods of locating soundings. The field party usually consists of two observers, a recorder, a leadsman, and boat crew. In tidal waters a tide-gauge reader is required. The observers, each with a sextant, occupy places in the sounding boat as close to the leadsman as practicable, in order that the observed position of each sounding may be very nearly the same as its true position. At the time a sounding is made, the two observers measure simultaneously the two angles between the lines of sight to three shore objects whose positions have been determined by the shore survey, one line of sight in each observation being directed toward the same object. The objects sighted to should be well defined and prominent, and so located with respect to the area to be sounded as to be readily visible from the sounding boat in all required positions. They should preferably be natural objects, such as church spires, windmills, lighthouses, cupolas, etc., but in case natural objects are not available, signal poles, such as are used for marking ranges, may be used. In making the observations, one observer measures the angular distance between the middle object and the object on the right; and the other observer measures the angular distance between the middle object and the object on the left. This is illustrated in Fig. 28, in which F represents the position of the boat at the time of a given sounding, and A and B are the two angles measured by the two observers between the lines of sight to the three shore objects E, C, and D. The sounding work is conducted as follows:

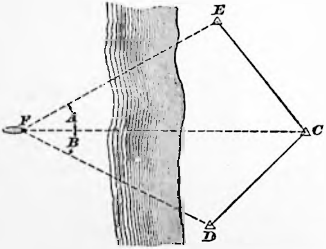

FIG. 28

The sounding boat moves slowly along a range or course, the leadsman making the soundings at the required intervals,

usually two or three per minute. A few seconds before a sounding is to be made the leadsman calls out "ready," when the observers hold their sextants in position to make an observation on the shore objects, each observer moving the vernier arm of his sextant so as to keep the two images in coincidence as the boat moves. At the moment the sounding is made, the leadsman calls out "sound," when each observer reads the angle on his sextant and calls it out to the recorder, who records each angle in its proper column in his notebook. The leadsman calls out the observed depth to the recorder, who also enters it in his notebook, together with the number of the sounding and the time the sounding is made. At required intervals, the leadsman observes the character of the bottom and informs the recorder, who enters it in the proper place in his notebook.

In some cases both angles are measured by one observer with a double sextant; they can also be measured successively by one observer with an ordinary sextant, if the boat is brought to a stop for each sounding. But they are usually measured by two observers, each using an ordinary sextant, in the manner just described. When sextants are used and the angles are so recorded by the recorder that the observer has only to observe and read them, four angles per minute can be observed under ordinary favorable conditions.

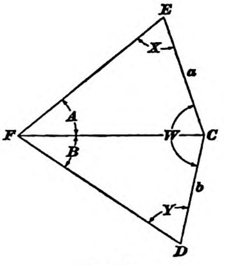

FIG. 29

41. The accurate location of soundings by two sextants involves what is known as the **three-point problem.** This problem can be solved trigonometrically as follows: Let F, Fig. 29, be the position of the boat when a given sounding is made; E, C, and D the three shore objects, whose positions are determined by the angle W and the sides EC and CD, which are designated by a and b, respectively. The angles A and B are the two sextant angles measured in the boat. The problem is to determine the distances FE and FD.

In the triangles EFC and CFD, Fig. 29 or 30,

$$CF = \frac{a \sin X}{\sin A} = \frac{b \sin Y}{\sin B} \qquad (a)$$

Also, $$X+Y+W+A+B=2\times180°=360°$$

Therefore, if S represents the sum of X and Y,

$$S=X+Y=360°-(W+A+B) \qquad (b)$$

and $$Y=S-X \qquad (c)$$

Then, $\sin Y=\sin (S-X)=\sin S \cos X-\cos S \sin X$

If this value of $\sin Y$ is substituted in equation (a),

$$\frac{a \sin X}{\sin A} = \frac{b\,(\sin S \cos X - \cos S \sin X)}{\sin B}$$

and, if the parenthesis is removed and the fractions are cleared, the equation becomes

$$a \sin X \sin B=b \sin S \cos X \sin A-b \cos S \sin X \sin A$$

If both sides of this equation are now divided by $\sin X$, $\dfrac{\cos X}{\sin X}$ is replaced by $\cot X$, and $\cot X$ is solved for, the equation obtained is

$$\cot X = \frac{a \sin B + b \cos S \sin A}{b \sin S \sin A} = \frac{a \sin B}{b \sin S \sin A} + \frac{\cos S}{\sin S}$$

Therefore, $$\cot X = \frac{a \sin B}{b \sin S \sin A} + \cot S \qquad (d)$$

The value of Y can be determined either similarly or by substituting the value of X in equation (c). After the values of X and Y are determined, the distances EF and DF can be found by trigonometry. In triangle EFC,

$$ECF = 180° - (A + X) \qquad (e)$$

$$EF = \frac{a \sin ECF}{\sin A} \qquad (f)$$

Similarly, in the triangle CFD,

$$DCF = 180° - (B + Y) \qquad (g)$$

$$DF = \frac{b \sin DCF}{\sin B} \qquad (h)$$

After the distances EF and DF for a given sounding are calculated, the position of the sounding can be located by drawing arcs with a pair of compasses, with E and D as centers and with radii of lengths EF and DF, respectively. The intersection of the two arcs will be the point F, or the

position of the sounding. When a rock, a reef, a buoy, or other important object is located by sextant angles to three fixed points on shore, the location should be calculated. However, the positions of soundings located by this method may be platted fairly accurately without calculation, as explained later.

EXAMPLE.—In Fig. 29, $a = 850$ feet, $b = 760$ feet, $W = 150°$, $A = 41° 30'$, and $B = 35°30'$. Calculate the distances $E F$ and $D F$.

SOLUTION.—If equation (*b*) is applied, $S = X + Y = 360° - (W + A + B) = 133°$. Then, by equation (*d*),

$$\cot X = \frac{850 \sin 35°30'}{760 \sin 133° \sin 41°30'} + \cot 133°$$

But, $\sin 133° = \sin (180° - 133°) = \sin 47°$, and $\cot 133° = -\cot (180° - 133°) = -\cot 47°$. Hence,

$$\cot X = \frac{850 \sin 35°30'}{750 \sin 47° \sin 41°30'} - \cot 47° = 1.34021 - 0.93252 = 0.40769$$

Then, $X = 67°49'$; and, by equation (*c*), $Y = 133° - 67°49' = 65°11'$.

By equations (*e*) and (*f*) for triangle $E F C$,

$$E C F = 180° - (41°30' + 67°49') = 70°41'$$

$$E F = \frac{850 \sin 70°41'}{\sin 41°30'} = 1{,}211 \text{ ft.} \quad \text{Ans.}$$

Also, by the equations (*g*) and (*h*) for triangle $C F D$,

$$D C F = 180° - (35° 30' + 65° 11') = 79° 19'$$

$$D F = \frac{760 \sin 79° 19'}{\sin 35° 30'} = 1{,}286 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. In Fig. 30, in which F is the position of the sounding boat at the time a given sounding is made, and E, C, D are the three shore points, let $a = 1{,}200$; $b = 965$; $W = 146° 30'$; $A = 28° 15'$; and $B = 22° 30'$; find the angles X and Y and the distances $E F$ and $D F$.

$$\text{Ans.} \begin{cases} X = 80° 21' \\ Y = 82° 24' \\ E F = 2{,}403 \\ D F = 2{,}437 \end{cases}$$

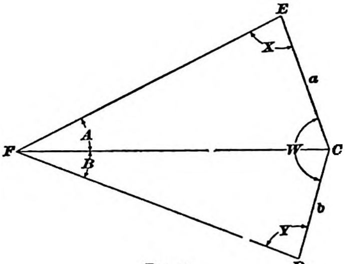

FIG. 30

2. With the same values for a, b, and W, let $A = 30°$

30′ and $B = 24° 28'$; find the angles X and Y and the distances EF and DF.

$$\text{Ans.} \begin{cases} X = 77° 3' \\ Y = 81° 29' \\ EF = 2{,}254 \\ DF = 2{,}240 \end{cases}$$

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42. By Transit and Stadia.—This is a rapid and efficient method of locating soundings in bodies of calm, smooth water and at distances that do not exceed the limit of good practice for stadia readings. In this method, the positions of the soundings are located with a transit on shore by means of observations taken on the stadia rod held in the sounding boat. Since the stadia rod should be without vertical motion when a reading is taken, it is evident that this method can be used satisfactorily only on smooth water. When this method for locating soundings is used, a complete hydrographic party comprises the observer on shore, with a transit equipped with stadia wires, and the boat party, consisting of the recorder, the leadsman, the stadiaman, and the boat crew. No signalman is required; the recorder acts as signalman. The soundings are not identified by time intervals, but by means of differently colored flags. A red flag is shown for every fifth sounding and a white flag for the intermediate soundings. Two general cases may occur under this method.

1. The transit station may be a point on the sounding range; in this case the azimuth of the range is known and each sounding is located by the distance, corresponding to the observed interval on the stadia rod, as measured along the range.

2. The transit station may be a point whose position has been determined but which is not on a sounding range; in this case the reading of the azimuth angle, as well as the stadia interval, must be observed and recorded for each sounding.

In either case the field work is conducted in the following manner: The sounding boat moves slowly along the range or course while the soundings are being made. The leadsman stands in the bow of the boat and makes the soundings,

calling out the observed depth of each sounding and also the character of the bottom at required intervals. The recorder enters in his notebook the number and the observed depth of each sounding, and the character of the bottom when noted by the leadsman. If the soundings are in tidal waters, the time should also be noted in order to make reductions for the tide heights as given in the notebook of the tide-gauge reader. During the sounding operations, the stadiaman holds the stadia rod vertical and facing the observer. He should be stationed close to the leadsman, in order that the observed positions of the soundings will coincide nearly with their true positions.

The observer keeps the vertical wire of the transit telescope directed to the stadia rod in the boat. If the transit station is on the range on which the soundings are made, the observer merely reads the stadia interval for each sounding and enters it in his notebook, also noting the time and the number of the sounding. If the transit station is not on the sounding range, but is off to one side, the vernier of the transit is first set at zero, the telescope is sighted on some object whose position has been determined, and the instrument is clamped. The upper plate is then unclamped, and the instrument turned in azimuth toward the sounding boat as explained for the second method. Then, for each sounding, the horizontal angle is read and recorded, in addition to the stadia interval, and the number and time of the sounding.

43. By a Fixed Line Marked by a Wire or Rope. This is an accurate method, but it is adapted only to narrow channels. It is often used for measuring cross-sectional areas in a canal or a small stream in connection with the determination of discharge or the measurement of material removed from the channel by dredging or other means. In such cases the wire or rope is stretched from bank to bank between fixed points, as illustrated in Fig. 31, and the soundings are taken at regular intervals along the wire or rope. The points where the soundings are taken are marked

by tin tags or by bits of cloth tied to the wire or rope. When this method is employed in connection with the measurement of dredged material, the stakes O, O' are carefully located and their positions noted, in order that they may be replaced if disturbed. Soundings are made at

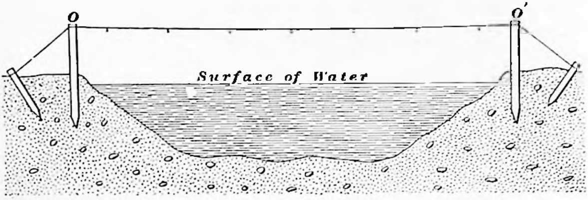


FIG. 31

known intervals along a wire or rope stretched from stake to stake, before and after the dredging operations.

44. By the Intersection of Fixed Ranges.—If a fixed range or section of considerable length is to be sounded a number of times, and the soundings are to be made at the same points each time, the soundings can be located by the

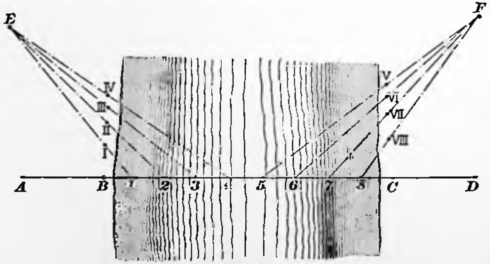


FIG. 32

intersection of a series of ranges with the fixed range or section. Let $A\,D$, Fig. 32, represent a range or section across a river, and *1*, *2*, *3*, *4*, *5*, *6*, *7*, *8* the points where soundings are to be made at successive periods. Range poles are set at A, B, C, and D, fixing the position of the range $A\,D$, unless

those points are marked by natural objects, and poles are also set at *E* and *F*; also at *I*, *II*, *III*, *IV*, etc. The ranges *E–I*, *E–II*, etc. and *F–V*, *F–VI*, etc., produced to their intersection with the range *A D*, will locate the soundings *1*, *2*, etc., and *5*, *6*, etc. The range signals *A*, *B*, *C*, *D*, and the back signals *E* and *F* for the numbered ranges may consist of ordinary range poles, whitewashed, as described in Art. **31**. The front range signals *I*, *II*, *III*, *IV*, *V*, etc., on the numbered ranges, should be designated by Roman numerals indicating the number of each range, as described in Art. **31**.

When this method is used, the sounding party consists of a recorder, a leadsman, and boat crew. The steersman keeps the boat on the fixed range *A D*, and each sounding is made when the leadsman, by sighting toward a numbered signal, finds himself in range with it and the back signal at the same time he is in line with the signals designating the range *AD*. Thus, when at the position *5*, Fig. 32, the leadsman is in line with the signals *CD*, designating the range *A D*, and also with the signals *VF*, designating range *V*. The boat can be stopped for each sounding if there is little or no current; or if the current is strong, the boat can move slowly, preferably against the current.

45. The Plane of Reference.—In making soundings, the leadsman notes the depth of each sounding below the water surface at the time the sounding is made. Since the elevation of the water surface is constantly changing, especially in tidal waters, it is necessary to select some particular stage or height of the water surface to which the depths of all the soundings are referred. Such a height of the water surface is called a **plane of reference**, and all observed depths are reduced to correspond to depths below this surface. In order to determine the proper reductions to apply to the soundings for different stages of the tide, it is necessary to know the elevation of the water surface at any given time during the sounding work; for this reason it is customary to employ a tide-gauge reader. In tidal waters, the surface of the water at mean low tide is usually taken as the

plane of reference, and the tide gauge should be set with the zero mark at the elevation of mean low tide. In such cases tide-gauge readings should be taken and recorded every 5 minutes during sounding operations, as has been explained. In lakes or reservoirs, where the elevation of the water surface changes but little and very slowly, the lowest recorded stage of the water is usually selected as the plane of reference. In such cases it is customary to take gauge readings twice a day—once in the morning and once in the afternoon during the period of sounding work. In rivers of variable stage, the plane of reference is usually the low-water stage of the river at the locality where the sounding work is done. In some cases, the general datum of the survey is used as a plane of reference. If the river is rising or falling rapidly while the soundings are being made, gauge heights should be read and recorded at intervals of 30 minutes, or even oftener if necessary. If, however, the river is at its normal stage during this period, and is changing but slowly, gauge readings should be taken twice a day, as for lakes or reservoirs.

FORMS FOR SOUNDING NOTES

46. Sounding Book.—For keeping the field notes of sounding work, three forms of field books are used; these are called, respectively, **sounding book**, **tide book**, and **angle book**. The sounding book, the form for which is shown in Form 1, is used by the recorder in the sounding boat. The first three columns contain, respectively, the number, the time, and the observed depth for each sounding; this information is obtained and entered by the recorder while in the sounding boat. The next two columns contain the reduction for tide and the reduced depth for each sounding; these are filled out in the office from the data obtained from the tide book, if the soundings are made in tidal waters. If the soundings are not made in tidal waters, no reductions are needed and these two columns are left blank. In the column headed Remarks should be entered the information obtained by the leadsman relative to the

character of the bottom, and also such information about the sounding ranges, the intervals between successive soundings, etc., as may be desirable. If a lead line is used in making the soundings, any errors in its length should be noted in this column, in order that proper corrections may be made to the observed depths.

FORM 1—SOUNDING BOOK

Soundings off Cape Charles
June 10, 1903

Johnson, Recorder
Kennedy, Leadsman

No.	Time	Soundings Feet	Reduction for Tide	Reduced Soundings Feet	Remarks
1	10:30	4.2			Range 13.
2		5.2			Soundings num-
3	10:31	8.2			bered west from
4		7.2			shore and made at
5	10:32	3.7			½-minute intervals.
6		3.3			Flag dropped at
7	10:33	3.8			1-minute intervals.
8		4.3			Bottom from sound-
9	10:34	2.8			ing No. 1 to 5, sand;
10		3.3			No. 6 to 8, shells.

47. Tide Book.—The tide book, shown in Form 2, is used by the tide-gauge reader; it contains the readings of the tide gauge at regular intervals of time and the time that each reading is taken. The direction and force of the wind also are usually noted and entered. From these notes, the proper reduction for tide can be obtained for each sounding, and the soundings can all be reduced to the plane of reference.

48. Angle Books.—Forms 3 and 4 show the form for the angle book that is used by the transit or sextant observer to record the angular measurements made in locating the

soundings. In the field he enters in the angle book the time and the observed angle for each sounding to be located. These are entered in the second and third columns, respectively, the first column being left blank until the observer obtains from the sounding book the numbers corresponding to the times of the observed soundings. Each observer is provided with an angle book in which he enters the field notes in the manner described. When the soundings are

FORM 2—TIDE BOOK

Observations of Tides at Cape Charles Gauge
June 10, 1903 J. Mason, Observer

Mean Time of Observation		Reading of Staff Gauge	Wind		Remarks
Hours	Min.	Feet	Direction	Force	
10	30	1.2	N W	Moderate	Gauge fastened to
10	35	1.3			pile at S. E. corner
10	40	1.4			of lighthouse wharf.
10	45	1.5			Tide rising.
10	50	1.6	W		Zero of gauge at
10	55	1.7			mean low water.
11	00	1.8			

located by time intervals and no angular measurements are made, the sounding book constitutes a complete office record of the soundings after the tide-gauge readings are obtained from the tide book and the reductions for the soundings have been entered.

49. **Office Record.**—When soundings are located by transit or sextant angles, a complete office record is obtained by combining the field notes in the manner shown in Form 5. The notes there shown are those given in Forms 1, 2, 3, and 4.

Form 3—Angle Book

Survey of Channel off Cape Charles, June 10, 1903
Observer No. 1, R. Briggs
Young & Sons' Transit No. 1612

No.	Time	Angle	Object Observed	Station Occupied	Remarks
1	10:30	41° 18′	Signal flag on launch	Transit Sta. A, at S.	Instrument set with vernier at
3	10:31	45° 00′		end of base line	zero when telescope points
5	10:32	49° 00′		on Cape Charles.	to Sta. B. Angles read to
7	10:33	54° 33′			the left from A–B. Sound-
9	10:34	61° 05′			ings on range 13, beginning
					at shore and running west.

FORM 4—ANGLE BOOK

Survey of Channel off Cape Charles, June 10, 1903
Observer No. 2, J. Smith,
Buff & Berger Transit No. 2840

No.	Time	Angle	Object Observed	Station Occupied	Remarks
1	10:30	43° 05′	Signal flag on launch	Sta. B on E. side of	Vernier at zero when telescope
3	10:31	49° 27′		entrance to bay,	points to Sta. A. Angles
5	10:32	57° 15′		opposite light-	read to right. Soundings on
7	10:33	64° 42′		house.	range 13, beginning near
9	10:34	73° 05′			shore and running west.

FORM 5—OFFICE RECORD

Survey of Channel off Cape Charles, June 10, 1903
C. F. Johnson, Recorder
T. Kennedy, Leadsman

Observer No. 1, Briggs, on Sta A, zeros on Sta B
Observer No. 2, Smith, on Sta. B, zeros on Sta. A

No.	Time	Soundings Feet	Reduced for Tide	Reduced Soundings Feet	Character of Bottom	Angles and Ranges		Remarks
						No. 1, Range 13	No. 2, Range 13	
1	10:30	4.2	1.2	3	Sand	41° 18′	43° 05′	
2		5.2	1.2	4				
3	10:31	8.2	1.2	7		45° 00′	49° 27′	
4		7.2	1.2	6				
5	10:32	3.7	1.2	2½	Sand	49° 00′	57° 15′	
6		3.3	1.3	2	Shells			
7	10:33	3.8	1.3	2½		54° 33′	64° 42′	
8		4.3	1.3	3	Shells			
9	10:34	2.8	1.3	1½		61° 05′	73° 05′	
10		3.3	1.3	2				

FORM 6

Survey of Midland Canal May 2, 1901				Johnson, Inst. Williams, Rod. — Jones, Taylor } Assistants — Gurley Level No. 2916
Sta.	Eleva.	Elevation of Water Surf	Remarks	Soundings
128	52.0	50.0	The water surface is used as the plane of reference.	$\frac{0.0}{2.0}$ $\frac{3.0}{5.0}$ $\frac{6.0}{10.0}$ $\frac{6.5}{13.0}$ $\frac{6.8}{18.0}$ $\frac{6.2}{28.0}$ $\frac{7.4}{38.0}$ $\frac{4.0}{44.0}$ $\frac{0.0}{48.0}$
129	52.5	50.0	Numerator equals the depth, in feet. Denominator equals the dist. from sta.	$\frac{0.9}{3.0}$ $\frac{3.0}{6.0}$ $\frac{6.5}{12.0}$ $\frac{7.0}{15.0}$ $\frac{7.4}{20.0}$ $\frac{6.3}{30.0}$ $\frac{7.0}{39.0}$ $\frac{3.0}{46.0}$ $\frac{0.0}{49.0}$
			Distances measured from left to right.	

50. Form 6.—When soundings are located by means of a wire or rope stretched across the stream, the field notes may be kept in the manner shown in Form 6. The notes there given represent sounding measurements across a canal at Stations 128 and 129 of the shore survey or traverse along the canal bank. The left-hand page of the notebook contains the number of the shore station in the first column, the ground elevation in the second column, and the elevation of the water surface in the third column. In the column headed Remarks is given all necessary information concerning the details of measurements, stage of water, etc. On the right-hand page of the notebook are given the sounding measurements. These are expressed in the form of fractions, the numerator designating the depth, in feet, and the denominator the distance from the shore station for each sounding. Thus, the fraction $\frac{6.0}{10.0}$ represents a depth of 6.0 feet at a distance of 10.0 feet from the shore station. The notes given in Form 6, when platted to scale on cross-section paper, show the cross-section of the canal for each station for which sounding notes are given.

PLATTING THE SOUNDINGS

51. General Methods.—Soundings are platted in various ways, according to the methods by which they are located. When located by ranges or courses, they are platted as follows: Each range or line on which soundings have been made is first platted to scale, in pencil, in the proper position on the map or chart. Then, the distances between the soundings on that range and the distance of each sounding from the end of the range being known, these distances are scaled on the pencil line and the position of each sounding as thus located is marked by a dot.

When the soundings are located by means of two transits on shore, as described in the third method, the base line is platted to scale in its proper position on the map and from each of its ends pencil lines are drawn making angles with the base equal to the angles measured in locating the soundings. The lines thus drawn represent the lines of

sight from the ends of the base to the positions of the soundings, and their directions are determined by the measured angles given in the notes. The intersection of corresponding lines drawn from the opposite ends of the base, representing the lines of sight of two simultaneous observations to a given sounding, will locate the sounding on the map.

In Fig. 33, *A B* represents a given base line as platted. The dotted lines *A–1*, *A–2*, *A–3*, and *B–1*, *B–2*, *B–3*, represent the directions of lines of sight to the soundings *1*, *2*, *3*. These lines are laid off at the observed angles as recorded in the notes. The intersections of the corresponding lines drawn from opposite ends of the base line give the locations of the respective soundings.

Soundings that have been located by this method can also be platted in the following manner: The distances from

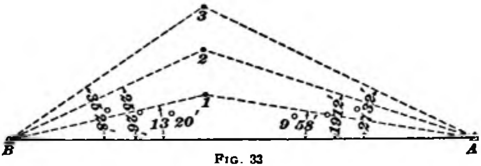
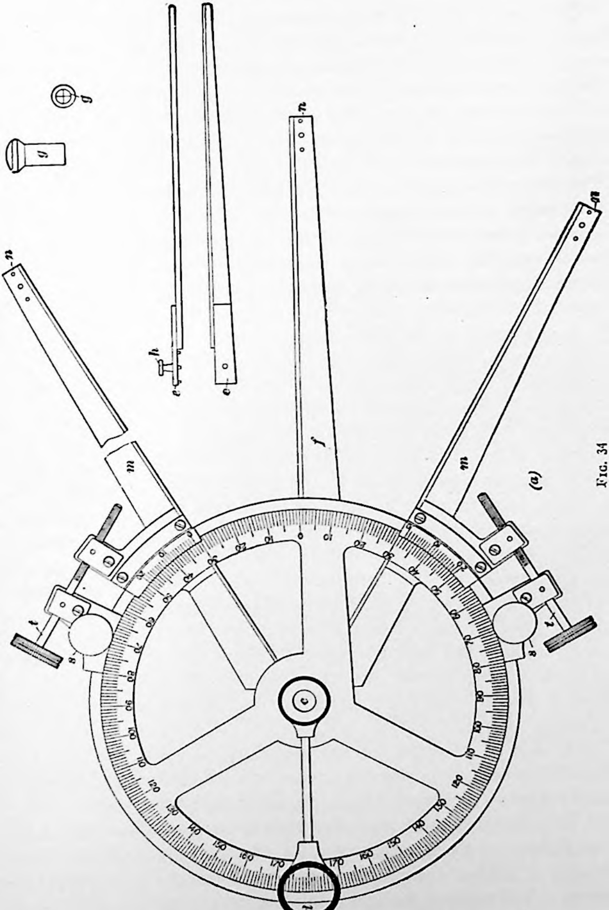


FIG. 33

each sounding to the respective extremities of the base line are calculated from the field notes. Then, the base line having been platted to scale in its proper position on the map, from its extremities as centers and with radii respectively equal to the two corresponding distances calculated for a given sounding, the arcs of two circles are drawn lightly in pencil with a pair of compasses. The intersection of the two arcs is the location of the sounding.

This method of platting is not nearly so expeditious as the method by the intersection of straight lines drawn from the instrument points, since the former method involves the calculation of two distances for each sounding, whereas the latter requires no calculation. It is sometimes valuable as a check, however, or to apply as a test in case of doubt regarding the position of a sounding.



(a)

Fig. 34

52. The Three-Arm Protractor.—If the soundings have been located by sextant observations, as described in the fourth method, the most convenient way to plat them is by the use of what is called a **three-arm protractor.**

This instrument, which is sometimes called a **station pointer,** consists of a graduated metal circle, to which is attached a fixed arm *f*, Fig. 34, and two movable arms *m*, *m*. The movable arms revolve around a central point, which is the center of the graduated circle. One edge of the fixed arm and the inner edges of the movable arms extend outwards radially in line with the central point, and are beveled, as shown. The circle is divided into 360 degrees, with the zero point opposite the beveled edge of the fixed arm, which is also known as the **zero arm,** so that the prolongation of this beveled edge passes through the zero mark on the circle and also through the center *c* of the instrument. Each movable arm is provided with a vernier, as shown in the figure, and also with a clamp screw *s* and tangent screw *t*. A magnifying glass *d* is pivoted and hinged to the center of the circle and swings parallel to the graduations.

Each instrument is usually provided with three interchangeable centers, which are cylindrical in form, as shown at *g*. Each center fits snugly into a cylindrical opening *c* in the center of the instrument. One center has a glass bottom, with two etched lines intersecting at the central point; another center has a transparent horn bottom with a small hole at the central point, through which a pencil point can be inserted; and the third center is provided with a spring needle point for pricking the central point into the drawing paper.

Three-arm protractors are made of several sizes. The graduated circle is usually from 5 to 6½ inches in diameter, and the arms are from 15 to 18 inches in length. Extensions for lengthening the protractor arms are furnished with each instrument. Each extension, as shown at *e*, *e*, Fig. 34, is provided with a splice to which are attached three studs that fit into corresponding holes at the end *n* of the protractor arm. After fitting the studs in place, the extension is secured

to the protractor arm by tightening the thumbscrew *h*. The extensions are used when soundings are to be platted that are beyond the reach of the regular protractor arms.

Before using a three-arm protractor, it is a good plan to carefully test the alinement and centering of the arms. To do this, place the protractor on the drawing board and draw lines along the straight edges of the three arms, then remove the protractor and prolong the lines inwards, noticing whether the three lines intersect in a common point. The operation should be repeated several times with the arms in different positions. If the three lines intersect in a common point for all positions of the arms, they may be considered to be truly centered.

53. The three-arm protractor is used almost exclusively for the purpose of platting soundings that have been located by sextant angles from the sounding boat. The way of using it is as follows: The movable arms of the protractor are set at the marks on the graduated circle designating the two sextant angles for any given sounding, and are firmly clamped. The instrument is then placed on the chart in such a position that the beveled edges of the three arms will pass through the platted positions of the three fixed points. This is done by placing the instrument on the paper with the beveled edge of the fixed or zero arm passing through the middle point, and sliding it around on the paper until the beveled edges of the two clamped arms also pass through the two respective outside points. The center of the instrument will then represent the position of the sounding. This point is marked by a pencil dot if a horn center is used, or pricked on the chart if a needle-point center is in the protractor at the time.

54. The Tracing-Paper Method.—When no three-arm protractor is available, soundings that have been located by sextant observations can be platted by means of a piece of tracing paper. Three lines are drawn on tracing paper in such positions as to intersect at a common point and include the two angles measured for any given sounding, the middle

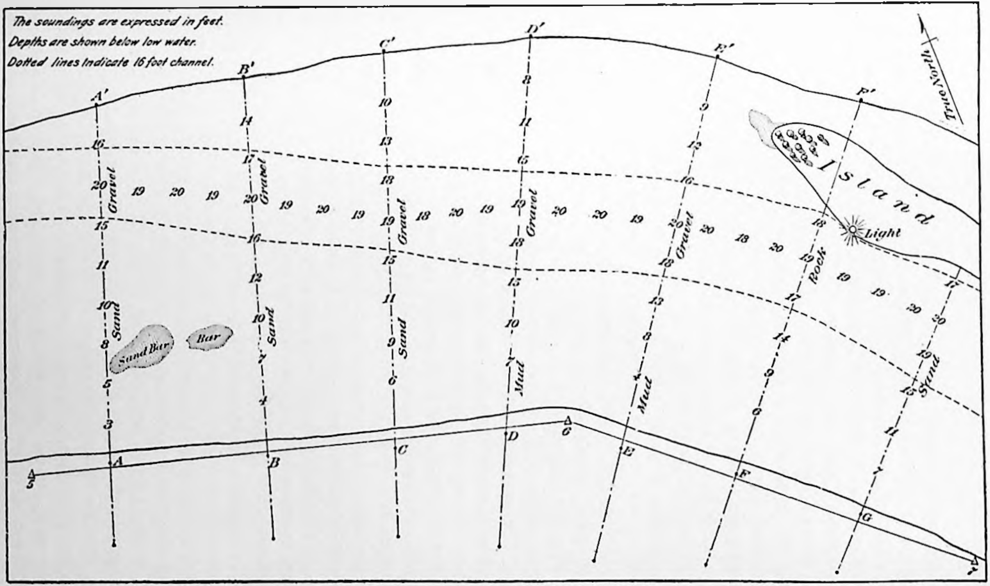
line forming a side of each angle. Then, to locate the sounding, the tracing paper is placed on the map in such a position that the three lines will pass through the platted positions of the three fixed points. The intersection of the three lines will then be the position of the sounding, which is pricked through the tracing on the map or chart.

HYDROGRAPHIC MAPS AND CHARTS

55. Maps or charts of hydrographic surveys should be drawn in accordance with the principles stated in *Mapping*, Parts 1 and 2. An outline map of a river or lake survey should show the lines and angles of the outline survey, and the triangulation stations, if any. It should also show the shore line and such details of the adjacent topography as may be considered necessary.

A complete hydrographic map of a river, lake, or reservoir should show, in addition to the outline of the water surface and the adjacent topography, the form or contour of the river bed or of the submerged portion of the containing valley or basin. In order to do this effectively, lines of equal depth should be drawn; these lines show the contours of the submerged area and correspond to contour lines on a topographical map. They are located and drawn on a hydrographic map in the following manner: The soundings are platted, the position of each sounding is indicated on the map by a small dot, and the depth of each sounding is written directly over its location on the map. The lines of equal depth or the subaqueous contours are then located and drawn according to the method described in *Mapping*, Part 2, for platting contours. The contour interval will vary according to the importance of the survey, the frequency of the soundings, and the object for which the survey is made.

56. Navigation Charts.—A navigation chart of a river, lake, harbor, or other navigable body of water should show, in addition to the shore line and the adjacent



The soundings are expressed in feet.
Depths are shown below low water.
Dotted lines indicate 16 foot channel.
True North
Island
Light
Sand Bar
Bar
Gravel
Sand
Mud
Rock
A'
B'
C'
D'
E'
F'
A
B
C
D
E
F
G
5
6
7

Fig. 35

topography, the position of the navigable channel and of all rocks, sand bars, reefs, sunken wrecks, or other obstacles to navigation. In a chart of a navigable river, it is customary to show both banks and such of the adjacent topography as is desired; also the positions of islands and of such obstacles to navigation as may exist. Contour lines are drawn showing the navigable channel and the outlines of sand bars and shoals. The depths of other parts of the stream or body of water are written in the positions where the soundings are made. The depths are usually expressed in feet, and are always so expressed in shallow water. In some cases fractional half feet are used, but not smaller fractions. Soundings in the sea where the depths exceed 18 feet are usually expressed in fathoms, a fathom being equal to 6 feet, but there is no uniform practice in the use of units for depth.

Fig. 35 represents a chart of a portion of a navigable river. The line of the survey is drawn in a light full line, and the angle stations are designated by numbers. The depths of soundings, in feet, are written in figures at the places where the soundings were made, and the character of the bottom is written under the figures expressing the depths. The limits of the navigable channel, which has a minimum depth of 16 feet, are shown by the dotted lines. The ranges on which the soundings were made are shown in the figure for purposes of illustration, but this is not customary in practice. In the field work of locating the soundings, the sounding ranges were laid out across the axis of the stream and range signals were established on both banks to fix the position of each range. The soundings were located by angles measured with a transit on shore, as described in Art. 38. The boat containing the sounding party started on the south side of the river and moved along range *A* to the north bank; the soundings were made at the required intervals as the boat progressed. After traversing range *A*, the boat proceeded to the north end of range *B*, and then moved southwards along that range while soundings were taken at regular intervals, until the depth of sounding indicated that the deepest part of the channel had been reached. Then, in

order to sound the deepest part of the channel, the boat was headed in a direction approximately at right angles to the range *B*, and propelled at a uniform speed toward the range *A*, soundings being made at regular intervals of time until the boat reached that range. The time taken to traverse the distance between the two ranges was noted, as was also the number of intermediate soundings and the time interval between soundings. Then the distance between the two ranges, the time taken by the boat to traverse this distance, the number of intermediate soundings, and the time interval between successive soundings all being known, the locations of the intermediate soundings were interpolated in the manner described in Art. **36**.

After the channel soundings between ranges *A* and *B* were made, the boat returned to range *B*, and sounding work on that range was resumed from the place indicated by signal from the transitman on shore. After range *B* was sounded, the boat proceeded to range *C* and moved northward along that range to the deepest part of the channel, then along the deep channel westwards to range *B*, soundings being made at regular intervals in the manner just described. The boat then returned to range *C* and proceeded northwards until the entire range had been sounded. It then proceeded to range *D* and sounded southwards along that range to the south bank, and so on back and forth across the river until all the ranges were sounded. Successive ranges were traversed in opposite directions and side trips were made for the channel soundings between each two adjacent ranges. For locating soundings made on ranges *A* and *B*, the observer was at the instrument point designated as Station 5, with the vernier of his transit at zero when the telescope was directed toward Station 6. After range *B* was sounded the transit was moved to Station 6 and set up over that station, from which soundings made on the remaining ranges were located. For locating soundings on ranges *C* and *D*, the vernier was set at zero when the telescope pointed to Station 5; and for locating soundings on ranges *E* and *F*, the vernier was set at zero when the telescope pointed to Station 7.

57. Chart of Harbor.—A navigation chart of the entrance to a harbor is shown in Fig. 36. The figures expressing depths, in feet, are written at the places where the soundings were made. Lines of equal depth are drawn at vertical intervals of 6 feet up to 18 feet, the depth of the navigable channel. Buoys marking the limits of the channel are shown in their proper positions along the 18-foot contour. The soundings in the southeastern part of the harbor, east of a line joining Station 3 and the buoy marked *H* were located by the intersections of transit lines observed from shore. In making the observations, the survey lines extending from Station 3 to Station 4 and from Station 4 to Station 5 were used successively for base lines. The buoys were located in the same manner, the transitmen occupying successive survey stations and using the survey lines extending between the stations as bases. The soundings farther from shore were located by sextant angles observed in the sounding boat to three fixed shore points; namely, the church, the lighthouse, and the windmill near Station 3.

MEASUREMENT OF VOLUME

CAPACITY OF A LAKE OR RESERVOIR

METHOD BY CONTOURS

58. Description of Method.—Different methods may be employed for determining the volume of water contained in a lake or reservoir, but the method by contour lines, about to be described, is probably the most accurate. An outline survey of the lake is made by traverse, stadia, or triangulation, as may be best suited to the case. The outline of the lake, as thus determined, is the surface contour. By means of soundings, the subaqueous contours of the containing valley or basin are determined at suitable intervals. The contour interval, or vertical distance between adjacent contours, is fixed according to the slopes of the valley or basin and the degree of accuracy required. The

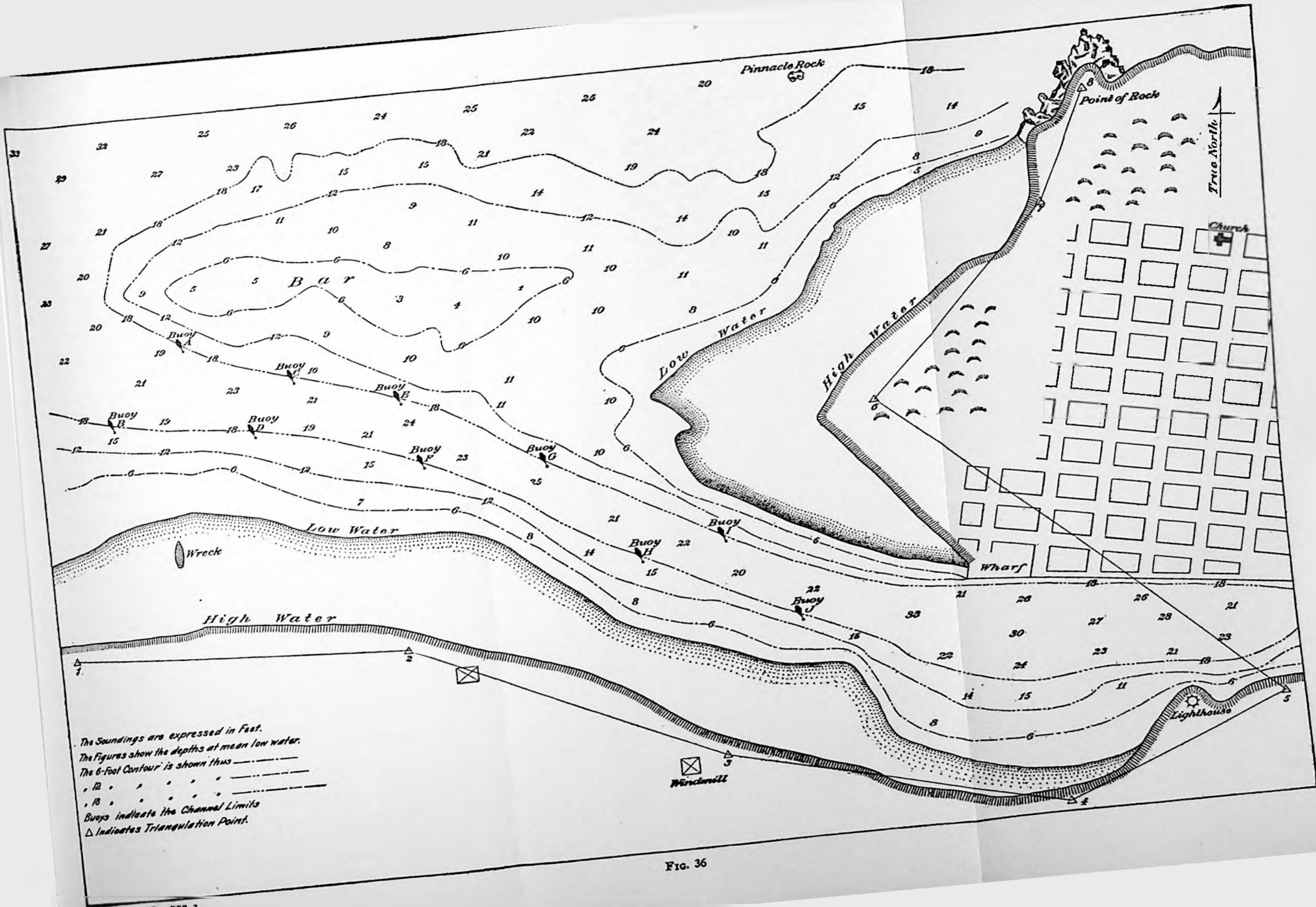

Pinnacle Rock
Point of Rock
True North
Church
Bar
Low Water
High Water
Buoy A
Buoy B
Buoy C
Buoy D
Buoy E
Buoy F
Buoy G
Buoy H
Buoy I
Buoy J
Wreck
Wharf
Lighthouse
Windmill
The Soundings are expressed in Feet.
The Figures show the depths at mean low water.
The 6-Foot Contour is shown thus
" 12 " " " "
" 18 " " " "
Buoys indicate the Channel Limits
△ Indicates Triangulation Point.

Fig. 36

notes thus obtained are then platted, and a map is made showing the outline of the water surface and the several contour lines.

The solid figure included between any two adjacent contours will resemble a prismoid, whose parallel end surfaces are the surfaces enclosed by the respective contour lines, and whose perpendicular length or height is the contour interval. The area of the water surface and the area enclosed by each contour line can be determined from the plat by any of the methods for finding the areas of irregular figures described in *Trigonometry*, Part 2. When the areas enclosed by the various contours, which form the end areas of the several prismoids, are known, the volume of each prismoid can be found approximately by multiplying one-half the sum of its end areas by its height. The sum of the volumes of the several prismoids will be the volume of water in the lake. This is what is known as the **end-area method** of calculating volumes.

59. Volume by End-Area Method.—Suppose that Fig. 37 represents the plat of a lake whose capacity is

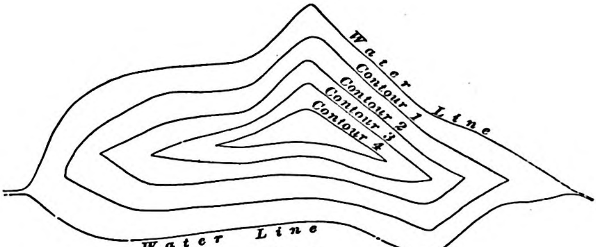


FIG. 37

required. Let A_0, A_1, A_2, A_3, and A_4 denote the areas bounded by the water-line and by the contours *1*, *2*, *3*, and *4*. respectively; let v_{0-1}, v_{1-2}, v_{2-3}, and v_{3-4} denote, respectively, the volumes of the prismoids included between the water surface and the first contour, and between the successive contours; also, let h denote the contour interval, and V the

total volume. By the method of average end areas, the approximate volumes of the several prismoids are

$$v_{0-1} = \frac{A_0 + A_1}{2} \times h$$

$$v_{1-2} = \frac{A_1 + A_2}{2} \times h$$

$$v_{2-3} = \frac{A_2 + A_3}{2} \times h$$

$$v_{3-4} = \frac{A_3 + A_4}{2} \times h$$

The total volume of the lake or reservoir is equal to the sum of the volumes of the several prismoids as expressed by the preceding equations, or

$$V = v_{0-1} + v_{1-2} + v_{2-3} + v_{3-4} = h\left(\frac{A_0}{2} + A_1 + A_2 + A_3 + \frac{A_4}{2}\right)$$

In order to express this as a general formula applicable to any number of contours, it may be written in the form

$$V = h\left(\frac{A_0}{2} + \Sigma A_m + \frac{A_n}{2}\right)$$

In this formula,

A_0 = area included by surface contour;
A_n = area included by lowest contour;
ΣA_m = sum of areas included by the intermediate contours.

Example.—Suppose that in Fig. 37 the contour interval is 5 feet and that the areas enclosed by the several contours are as follows: A_0 = 13,350 square feet, A_1 = 8,100 square feet, A_2 = 4,280 square feet, A_3 = 1,925 square feet, and A_4 = 520 square feet; find the volume of water in the lake, in cubic feet, by the end-area method.

Solution.—By substituting the given values in the formula, the volume of water is found to be

$$V = 5 \times \left(\frac{13,350}{2} + 8,100 + 4,280 + 1,925 + \frac{520}{2}\right) = 106,200 \text{ cu. ft. Ans.}$$

60. Volume by Prismoidal Formula.—If the volumes of the prismoids are calculated by the prismoidal formula, two adjacent prismoids are taken as one prismoid whose height is equal to twice the contour interval. The area included by the contour that lies between the two prismoids

taken is considered the middle area of the prismoid and so used in tne formula. By thus combining the first two prismoids of Fig. 37 and applying the prismoidal formula given in *Geometry*, Part 2, the expression for their volume is

$$v_{0-1} + v_{1-2} = \frac{2h}{6}(A_0 + 4A_1 + A_2)$$

The volume of the last two prismoids is

$$v_{2-3} + v_{3-4} = \frac{2h}{6}(A_2 + 4A_3 + A_4)$$

By adding these two expressions, the total volume of the lake is found to be

$$V = v_{0-1} + v_{1-2} + v_{2-3} + v_{3-4}$$
$$= \frac{h}{3}(A_0 + 4A_1 + 2A_2 + 4A_3 + A_4)$$

In order to express this as a general formula applicable to any number of contours, it may be written in the form

$$V = \frac{h}{3}(A_0 + 4\Sigma A_1 + 2\Sigma A_2 + A_n)$$

In this formula,

A_0 = area included by surface contour;
A_n = area included by lowest contour;
ΣA_1 = sum of areas included by intermediate contours whose subscripts are odd numbers;
ΣA_2 = sum of areas included by intermediate contours whose subscripts are even numbers.

EXAMPLE.—Suppose that in Fig. 37 all values are the same as in the example solved in Art. 59; namely, h = 5 feet, A_0 = 13,350 square feet, A_1 = 8,100 square feet, A_2 = 4,280 square feet, A_3 = 1,925 square feet, and A_4 = 520 square feet; what is the volume of water in the lake, in cubic feet, as determined by the prismoidal formula?

SOLUTION.—By substituting the given values in the formula,

$$V = \frac{5}{3}(13{,}350 + 4 \times 8{,}100 + 2 \times 4{,}280 + 4 \times 1{,}925 + 520)$$
$$= 104{,}217 \text{ cu. ft. Ans.}$$

61. Construction for Interpolating Contour.—It is evident that the prismoidal formula can be applied to the prismoids in pairs, as just described, only when there is an even number of prismoids. When there is an odd number

of prismoids, the last prismoid may be computed separately by the method of average end areas, or by interpolating a middle contour on the contour map, calculating the area included by it, and then applying the prismoidal formula. The middle contour can be interpolated as follows: The two end contours are platted to the same scale, preferably on cross-section paper, the smaller inside the larger making the two figures concentric as nearly as practicable. A third contour is then drawn in such position that each point will be midway between the corresponding points of the inner and outer figures. This interpolated contour can be sketched in largely by the eye, aided to such an extent as may be desired by measurements on lines drawn radially from a point approximately in the center of the figure. The area of the surface included by this interpolated contour may be taken as the middle area of the prismoid. This area can be easily determined by means of the cross-section paper or by the planimeter.

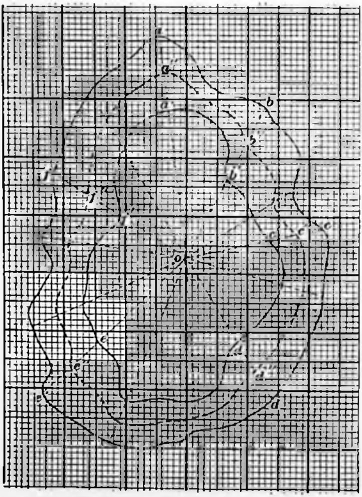


FIG. 38

Thus, in Fig. 38, $a\,b\,c\,d\,e\,f$ and $a'\,b'\,c'\,d'\,e'\,f'$ represent the contours including, respectively, the upper and the lower base of a prismoid, as platted on cross-section paper. The contour represented by the dotted line $a''\,b''\,c''\,d''\,e''\,f''$ lies midway between the boundaries of the two bases. This contour

is constructed or interpolated in the following manner: The point o is chosen as the center and the radial lines $o\,a, o\,b, o\,c$, etc. drawn to the outer contour; a', b', c', etc. are the points where these lines cross the inner contour. By measurement, the point a'' is located on the radial line $o\,a$ midway between a and a'; in like manner, b'' is located midway between b and b'; c'' is located midway between c and c', etc. The contour is then sketched through the points a'', b'', c'', d'', e'', and f'', as represented by the dotted line. The area of the surface enclosed by this interpolated contour is then determined by counting the squares of the cross-section paper, and is taken as the middle area of the prismoid.

EXAMPLES FOR PRACTICE

1. Suppose that the areas bounded by the water-line of a lake and by contours 1, 2, 3, 4, and 5 are as follows: $A_0 = 15{,}450$ square feet, $A_1 = 10{,}240$ square feet, $A_2 = 8{,}360$ square feet, $A_3 = 7{,}730$ square feet, $A_4 = 6{,}890$ square feet, and $A_5 = 5{,}240$ square feet. If the contour interval is 10 feet, calculate the volume of water in the lake, in cubic feet, by the end-area method. Ans. 435,650 cu. ft.

2. Suppose that the areas bounded by the water-line of a lake and by contours 1, 2, 3, 4, 5, and 6 are as follows: $A_0 = 14{,}320$ square feet, $A_1 = 10{,}280$ square feet, $A_2 = 9{,}360$ square feet, $A_3 = 7{,}480$ square feet, $A_4 = 5{,}780$ square feet, $A_5 = 4{,}760$ square feet, and $A_6 = 3{,}250$ square feet. If the contour interval is 5 feet, calculate the volume of water in the lake by the prismoidal formula. Ans. 229,880 cu. ft.

METHOD BY PARALLEL CROSS-SECTIONS

62. Description of Method.—The following is also a good method for determining approximately the capacity of a lake or reservoir: A survey is made to determine the outline of the water surface, which is platted accurately to scale. Then, at selected points, parallel ranges are laid out across the lake, dividing its surface into trapezoids, as illustrated in Fig. 39. If the shores of the lake are irregular, the ranges are so located that straight lines connecting the points where the adjacent ranges intersect the shore line will be as much inside as outside of the water-line. By the

aid of the plat this can usually be done with a reasonable degree of accuracy. The small irregular areas included between the straight line and the water-line will then approximately balance, and it will be sufficiently accurate to consider the lake boundary as straight between adjacent ranges. The ranges having been located, soundings are made along them and the cross-section of the lake is determined on each range. The cross-sections are platted, as shown in Fig. 39, and the area of each cross-section is computed in the following manner: The

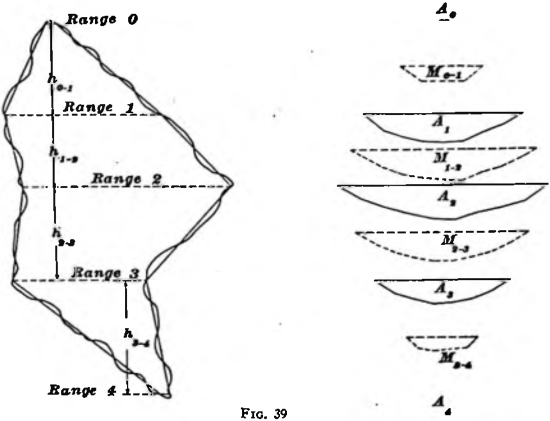


FIG. 39

section is first imagined to be divided into a series of trapezoids and triangles by vertical lines through the points at which soundings are taken. Then the area of each such trapezoid and triangle is determined, and these areas are added together to give the area of the entire section.

The lake basin is thus divided into prismoids, whose bases are the cross-sections, and whose altitudes are the perpendicular distances between adjacent ranges, and the capacity of the lake is equal to the sum of the volumes of the prismoids.

63. Volume by End-Area Method.—The approximate capacity of the lake or reservoir can be calculated by the

method of average end areas in the following manner: Let A_0, A_1, A_2, A_3, and A_4 denote, respectively, the areas of the cross-sections on the parallel ranges designated in Fig. 39 as *Range 0*, *Range 1*, *Range 2*, *Range 3*, and *Range 4*, respectively. Also, let h_{0-1}, h_{1-2}, h_{2-3}, and h_{3-4} denote, respectively, the perpendicular distances between the adjacent parallel ranges *1*, *2*, *3*, and *4*, as shown in the figure, and let v_{0-1}, v_{1-2}, v_{2-3}, and v_{3-4} denote the volumes of the corresponding prismoids. The end range *0* is merely a short straight line that represents the end of the lake and corresponds somewhat to the cutting edge of a wedge in the prismoid included between it and the cross-section on the adjacent range *1*. The same is true of the end range *4* with respect to the prismoid included between it and the cross-section on range *3*. The cross-sections on *0* and *4* thus consist of a straight line merely, and each of the areas A_0 and A_4 is zero. These areas should therefore be taken at zero in computing the volumes of the two end prismoids. By the end-area method, the expressions for the volumes of the several prismoids are: $v_{0-1} = \frac{A_0 + A_1}{2} \times h_{0-1}$; $v_{1-2} = \frac{A_1 + A_2}{2} \times h_{1-2}$; $v_{2-3} = \frac{A_2 + A_3}{2} \times h_{2-3}$; $v_{3-4} = \frac{A_3 + A_4}{2} \times h_{3-4}$.

Hence, the total volume of the lake is

$$V = v_{0-1} + v_{1-2} + v_{2-3} + v_{3-4} = \tfrac{1}{2}\left[(A_0 + A_1)h_{0-1} + (A_1 + A_2)h_{1-2} + (A_2 + A_3)h_{2-3} + (A_3 + A_4)h_{3-4}\right]$$

This formula applies to Fig. 39 or to any lake or reservoir for which the measurements are made on five parallel ranges; its application to such a case will be clearly understood. In order to make the formula applicable to measurements made on any number of ranges, it may be written in the form

$$V = \tfrac{1}{2}\left[(A_0 + A_1)h_{0-1} + (A_1 + A_2)h_{1-2} \ldots . (A_m + A_n)h_{m-n}\right]$$

In this formula, A_n denotes the area of the cross-section on the last range, and A_m that on the next to the last range.

EXAMPLE.—Suppose that the areas of the several cross-sections of the lake shown in Fig. 39, as measured along the ranges, are: $A_0 = 0$,

$A_1 = 4{,}256$ square feet, $A_2 = 6{,}322$ square feet, $A_3 = 3{,}130$ square feet, and $A_4 = 0$; also, that the perpendicular distances between ranges are: $h_{0-1} = 250$ feet, $h_{1-2} = 192$ feet, $h_{2-3} = 256$ feet, and $h_{3-4} = 310$ feet; what is the capacity of the lake, in cubic feet, as calculated by the end-area method?

SOLUTION.—By substituting the given values in the formula, the operations, in detail, are as follows:

$$
\begin{array}{rcll}
(A_0 + A_1)h_{0-1} &=& (0 + 4{,}256) \times 250 &= \;\;1\,0\,6\,4\,0\,0\,0 \\
(A_1 + A_2)h_{1-2} &=& (4{,}256 + 6{,}322) \times 192 &= \;\;2\,0\,3\,0\,9\,7\,6 \\
(A_2 + A_3)h_{2-3} &=& (6{,}322 + 3{,}130) \times 256 &= \;\;2\,4\,1\,9\,7\,1\,2 \\
(A_3 + A_4)h_{3-4} &=& (3{,}130 + 0) \times 310 &= \;\;\;\;\underline{9\,7\,0\,3\,0\,0} \\
 & & & \;\;2\overline{)6\,4\,8\,4\,9\,8\,8} \\
 & & V &= \;\;3\,2\,4\,2\,4\,9\,4 \text{ cu. ft. Ans.}
\end{array}
$$

64. Interpolating Middle Cross-Section.—When the volume of a lake or reservoir is determined by measuring cross-sections on parallel ranges, the ranges cannot usually be located advantageously at uniform intervals, but must be located in such positions as will determine most accurately the form of the lake or reservoir. Consequently, two adjacent prismoids cannot be considered as one prismoid whose length is equal to the aggregate length of the two, and the intervening section considered as the middle section in applying the prismoidal formula, as in the preceding method. For, since the two prismoids are not of the same length, the intervening section is not the middle section. Hence, when the prismoidal formula is applied to this method, the middle area of each prismoid is determined by platting its two end cross-sections together on cross-section paper and interpolating a middle cross-section. The area of the interpolated cross-section is then determined and taken as the middle area of the prismoid. The method of interpolating the middle cross-section is similar to that for interpolating the middle contour explained in Art. 61, but is even more simple.

Let *L M N* and *O P Q*, Fig. 40, represent the bottom profiles of the measured cross-sections on two adjacent ranges. A third line *R S T* is drawn in such position that each point is midway between corresponding points in the other two lines. In most cases, it will be sufficiently accurate to locate

points on the interpolated line midway between corresponding points that are located by soundings on the other two lines. Thus, the point S is located midway between M and P, which are points that have been located by soundings. The point R is located in the horizontal surface line midway between O and L, and the point T in the surface line midway between N and Q. The surface lines of the three cross-sections coincide between L and N. The area of the interpolated cross-section can now be determined by counting the squares of the cross-section paper, and this is taken as the middle area of the prismoid whose end areas are the areas of the cross-sections LMN and OPQ. The middle area of the two end prismoids is determined in a similar manner. In this case the end cross-section is a straight line, and points

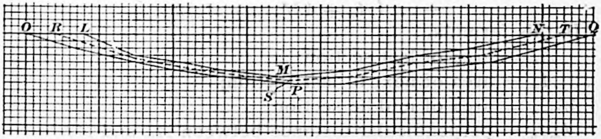


FIG. 40

on the interpolated line are located midway between the points that have been located by soundings and corresponding points on the straight line, at proportional distances from the ends of the line. When the volume of a lake or reservoir is determined by measuring cross-sections on parallel ranges and applying the prismoidal formula, the middle area of each prismoid is determined in the manner described.

65. Volume by Prismoidal Formula.—When the area of the middle cross-section has been determined, the volume of each prismoid can be determined by applying the prismoidal formula in the usual manner. The sum of the volumes of the several prismoids is the volume of water contained in the lake or reservoir.

In Fig. 39, let M_{0-1}, M_{1-2}, M_{2-3}, and M_{3-4} denote the middle areas of the prismoids whose altitudes are h_{0-1}, h_{1-2}, h_{2-3}, and h_{3-4}, respectively. Then, by applying the prismoidal

formula, the volumes of the several prismoids are found to be as follows:

$$v_{0-1} = \frac{h_{0-1}}{6}(A_0 + 4M_{0-1} + A_1)$$

$$v_{1-2} = \frac{h_{1-2}}{6}(A_1 + 4M_{1-2} + A_2)$$

$$v_{2-3} = \frac{h_{2-3}}{6}(A_2 + 4M_{2-3} + A_3)$$

$$v_{3-4} = \frac{h_{3-4}}{6}(A_3 + 4M_{3-4} + A_4)$$

Then, $V = v_{0-1} + v_{1-2} + v_{2-3} + v_{3-4}$,

or $V = \frac{1}{6}[(A_0 + 4M_{0-1} + A_1)h_{0-1} + (A_1 + 4M_{1-2} + A_2)h_{1-2} + (A_2 + 4M_{2-3} + A_3)h_{2-3} + (A_3 + 4M_{3-4} + A_4)h_{3-4}]$

In order to express this as a general formula applicable to any number of cross-sections, it may be written in the form

$$V = \tfrac{1}{6}[(A_0 + 4M_{0-1} + A_1)h_{0-1} + (A_1 + 4M_{1-2} + A_2)h_{1-2} + \ldots (A_m + 4M_{m-n} + A_n)h_{m-n}]$$

In this formula,

A_n = area of last section;

A_m = area of next to last section;

M_{m-n} = area of middle section;

h_{m-n} = perpendicular distance between A_m and A_n.

EXAMPLE.—Suppose that all values are the same as in the example solved in Art. 63; namely, $A_0 = 0$, $A_1 = 4{,}256$ square feet, $A_2 = 6{,}322$ square feet, $A_3 = 3{,}130$ square feet, $A_4 = 0$, $h_{0-1} = 250$ feet, $h_{1-2} = 192$ feet, $h_{2-3} = 256$ feet, and $h_{3-4} = 310$ feet; and suppose, also, that the areas of the interpolated middle sections are: $M_{0-1} = 1{,}107$ square feet, $M_{1-2} = 5{,}498$ square feet, $M_{2-3} = 4{,}536$ square feet, and $M_{3-4} = 863$ square feet; what is the capacity of the lake, in cubic feet, as calculated by the prismoidal formula?

SOLUTION.—By substituting the given values in the formula, the operations, in detail, are as follows:

$(A_0 + 4M_{0-1} + A_1)h_{0-1} = (0 + 4 \times 1{,}107 + 4{,}256) \times 250 \ldots = 2171000$

$(A_1 + 4M_{1-2} + A_2)h_{1-2} = (4{,}256 + 4 \times 5{,}498 + 6{,}322) \times 192 \ldots = 6253440$

$(A_2 + 4M_{2-3} + A_3)h_{2-3} = (6{,}322 + 4 \times 4{,}536 + 3{,}130) \times 256 \ldots = 7064576$

$(A_3 + 4M_{3-4} + A_4)h_{3-4} = (3{,}130 + 4 \times 863 + 0) \times 310 \ldots\ldots = 2040420$

6)17529436

$V = 2921573$ cu. ft. Ans

EXAMPLES FOR PRACTICE

1. In Fig. 41, which represents a lake, BC, DE, and FG are parallel ranges on which soundings have been taken. The depths of

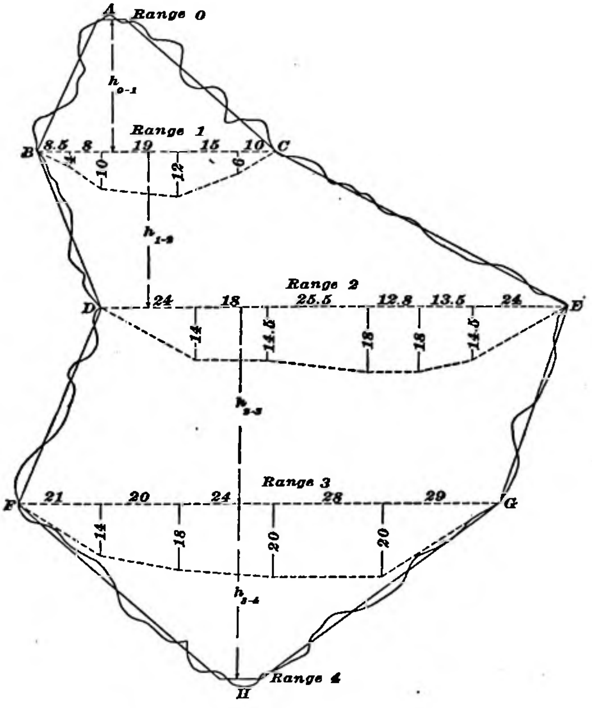


FIG. 41

the soundings and the distances between soundings are indicated in the figure. Using these values, calculate the areas A_1, A_2, and A_3 of the cross-sections on the ranges.

$$\text{Ans.}\begin{cases} A_1 = 447.0 \text{ sq. ft.} \\ A_2 = 1{,}462.7 \text{ sq. ft.} \\ A_3 = 1{,}773.0 \text{ sq. ft.} \end{cases}$$

2. If, in Fig. 41, the distances between adjacent ranges are $h_{0-1} = 35$ feet, $h_{1-2} = 42$ feet, $h_{2-3} = 54$ feet, and $h_{3-4} = 48$ feet, what is the capacity of the lake as determined by the method of average end areas? Ans. 177,840 cu. ft.

3. Plat the cross-sections shown in Fig. 41 to a horizontal scale of 1 inch = 20 feet and a vertical scale of 1 inch = 10 feet.

4. In Fig. 41, the areas, in square feet, of the middle sections of the various prismoids are found to have the following values: Between ranges *0* and *1*, $M_{0-1} = 131.4$; between ranges *1* and *2*, $M_{1-2} = 856.3$; between ranges *2* and *3*, $M_{2-3} = 1{,}626.7$; between ranges *3* and *4*, $M_{3-4} = 487.3$. What is the capacity of the lake, according to the prismoidal formula? Ans. 160,480 cu. ft.

CAPACITY OF A VALLEY OR BASIN FOR WATER STORAGE

66. By Contours.—If close results are desired, it is best to make a complete topographical survey of the area to be flooded and construct a contour map of the area, employing the methods described in *Topographic Surveying*. The general method is as follows: The location of the dam for impounding the water having been selected, the elevation of the spillway or overflow is decided on; this determines the height of the water in the basin. The **spillway** is that part of the dam over which the waste water is allowed to flow, and is usually somewhat lower than the crest of the dam. The impounded water will rise to a height corresponding to the elevation of the spillway and will form a pond or lake whose boundary will be a contour line extending around the border of the basin. This line, whose position thus defines the limits of the area overflowed by the water, is called the **flow line**. In Fig. 42 the flow line, which is at the elevation of the spillway, is one contour interval lower than the crest of the dam.

After the survey has been made, the flow line, the successive contours, and the outline of the projected dam are platted, as shown in Fig. 42. The planes of the contours, including that of the flow line, will intersect the face of the dam in a series of horizontal lines, as shown. A line joining the ends of these horizontal lines on the inner face of the dam, at the points where they meet the sides of the valley,

will indicate the inner outline of the base of the dam, or the inner toe of the slope, as shown in Fig. 42. A similar line will indicate the outer toe of the slope. Only the contour lines and flow lines need be considered, however, in determining the capacity of the basin. The areas enclosed by the flow line and the several contour lines are either calculated or measured with a planimeter, and the capacity of the reservoir is determined by the method of average end areas or by the prismoidal formula, as explained in preceding articles.

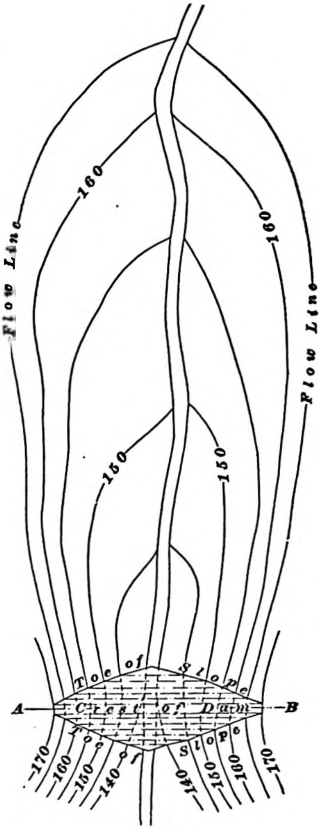


FIG. 42

EXAMPLE.—Suppose that in Fig. 42 the contour interval is 5 feet and that the areas enclosed by the several contours are as follows: $A_0 = 9{,}475$ square feet, $A_1 = 7{,}415$ square feet, $A_2 = 4{,}175$ square feet, $A_3 = 1{,}810$ square feet, and $A_4 = 685$ square feet; what is the capacity of the reservoir, in cubic feet, as determined: (*a*) by the end-area method? (*b*) by the prismoidal formula?

SOLUTION.—(*a*) Substituting the given values in the formula in Art. 59,

$$V = 5\left(\frac{9{,}475}{2} + 7{,}415 + 4{,}175 + 1{,}810 + \frac{685}{2}\right) = 92{,}400 \text{ cu. ft.} \quad \text{Ans.}$$

(*b*) Substituting the given values in the formula in Art. 60,

$$V = \tfrac{5}{3}[9{,}475 + 4(7{,}415 + 1{,}810) + 2 \times 4{,}175 + 685] = 92{,}350 \text{ cu. ft.}$$

EXAMPLES FOR PRACTICE

1. From a survey made of a reservoir to determine its capacity, the areas enclosed by the flow line and the four successive contours are found to be as follows: $A_0 = 4{,}095$ square feet, $A_1 = 3{,}156$ square feet, $A_2 = 2{,}369$ square feet, $A_3 = 1{,}854$ square feet, and $A_4 = 1{,}044$ square feet; if the contour interval is 4 feet, what is the capacity of the reservoir as determined by the method of average end areas?

Ans. 39,794 cu. ft.

2. What is the capacity of the reservoir referred to in the preceding example, as determined by the prismoidal formula? Ans. 39,889 cu. ft.

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67. By Parallel Cross-Sections.—The capacity of a valley or basin for water storage can also be determined approximately as follows: A site having been selected for a dam, the elevation of the spillway is fixed and a survey is made to determine the location of the flow line on the ground. A plat of the survey is made showing the flow line and the outline of the projected dam. Suitable locations are then selected for a series of parallel cross-lines joining points on the flow line, situated on opposite sides of the valley. These cross-lines are located in such positions as to divide the area enclosed by the flow line into trapezoids. The cross-lines are located in such positions that straight lines joining the ends of adjacent cross-lines will either coincide with the flow line or lie equally on both sides of it. This can usually be done easily by the aid of the plat even when the flow line is quite irregular.

The locations for the parallel cross-lines having been determined from the plat, the lines are located and measured on the ground, and levels are taken over them, thus determining the cross-section of the valley within the flow line on each of the parallel cross-lines. The profile of this cross-section is platted, preferably on cross-section paper, and a straight line is drawn joining the points where the profile intersects the flow line. This straight line is horizontal and corresponds to what will be the water surface when the basin or reservoir is full of water to the flow line, and the cross-section thus formed represents what will be the cross-section

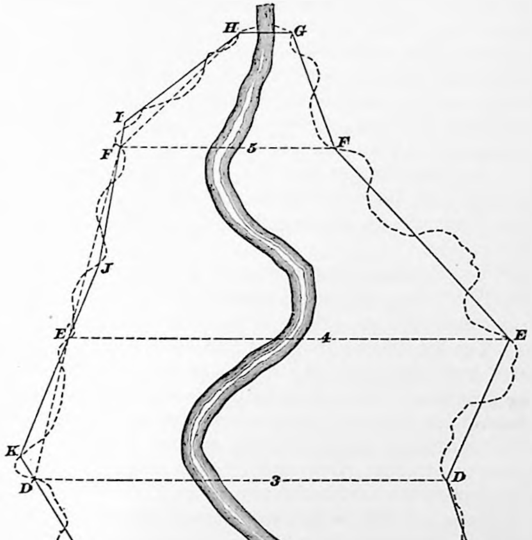

FIG. 43

of the water. The several cross-sections thus determined correspond to the cross-sections of a lake as obtained by soundings. The perpendicular distances between adjacent cross-sections are measured on the ground, calculated from the outline survey or scaled from the plat. The area of each cross-section is found, and the volume of each prismoid and total volume of the lake are determined by the end-area method as described in Art. **63**, or by the prismoidal formula, as in Art. **65**. In the latter case, the middle section may be interpolated, as described in Art. **64**, or measured on the ground.

Let Fig. 43 represent the plat of a closed survey around the limits of the reservoir, following the approximate position of the flow line. Such a position is obtained for the survey line by following along the side of the valley and locating each station of the survey at or near the elevation of the flow line. The irregular line is the flow line. AB is the axis of the dam, and the dotted lines *1*, *2*, *3*, etc. are the parallel cross-lines that divide the area enclosed by the flow line into trapezoids. One end of each cross-line is located at a station of the outline survey, as at C, D, E, F, or M, but the opposite end of the line will not usually fall at a station, but at some intermediate point on the line, as at C', D', E', F', or M'. If straight lines are drawn joining such ends of the cross-lines as lie at intermediate points on the survey line with the ends of the adjacent cross-lines, the lines so drawn and the corresponding parts of the survey line will form small triangles, some of which will lie inside, and some outside, of the original outline survey. Thus, the line $D'E'$ forms one side of the small triangle $D'E'K$; the line $E'F'$ forms one side of the small triangle $E'JF'$, etc. The effect of such triangles is usually small, however, and in most cases it is possible to locate the cross-lines in such positions that straight lines joining the corresponding ends of adjacent cross-lines will approximate the flow line sufficiently closely, and the small triangles formed may be neglected.

The lines BC, CD, DE, EF, and FG are sides of the survey, and it is seen from the plat that they cut off small

irregular areas that lie between the survey line and the flow line on both sides of the latter, and that these areas will approximately balance; that is, the areas lying on the outside of the survey line will be approximately equal to those lying inside of it. Similarly, if the points H and F', F' and E', E' and D', etc. are joined by straight lines, these lines will cut off approximately equal areas on both sides of the flow line. The trapezoid $F' H G F$ will then contain approximately the same area as the irregular figure $F' I H G F$ bounded by the flow line; and similarly, each trapezoid included between any two adjacent parallel cross-lines will be approximately equal to the figure included between the same two cross-lines and limited by the flow line. The valley or basin is thus divided into prismoids whose bases are the cross-sections measured on the parallel cross-lines, and whose altitudes are the perpendicular distances between adjacent cross-lines. From these the volume of each prismoid can be calculated as explained in Art. 65. The capacity of the valley or basin is the sum of the volumes of the several prismoids.

EXAMPLES FOR PRACTICE

1. The areas of the several cross-sections of a valley intended for water storage are found to be: $A_0 = 0$, $A_1 = 396$ square feet, $A_2 = 678$ square feet, $A_3 = 910$ square feet, $A_4 = 720$ square feet, and $A_5 = 586$ square feet. The perpendicular distances between the cross-sections are $h_{0-1} = 40$ feet, $h_{1-2} = 45$ feet, $h_{2-3} = 50$ feet, $h_{3-4} = 32$ feet, and $h_{4-5} = 36$ feet. What will be the capacity of the reservoir, in cubic feet, as determined by the end-area method? Ans. 121,373 cu. ft.

2. Assuming the middle areas to be: $M_{0-1} = 280$ square feet, $M_{1-2} = 410$ square feet, $M_{2-3} = 802$ square feet, $M_{3-4} = 830$ square feet, and $M_{4-5} = 670$ square feet, calculate, by the prismoidal formula, the capacity of the reservoir referred to in the preceding example.

Ans. 120,744 cu. ft

METHODS FOR HYDROGRAPHIC SURVEYING ALONG COAST

METHODS FOR SOUNDING

68. Reasons for Special Methods for Hydrographic Surveying.—In many localities along the two coasts of the United States, the U. S. Coast and Geodetic Survey extends hydrographic surveys out into the ocean to a depth of 6,000 feet, or 1,000 fathoms. The fathom is the more common unit for measuring soundings made on the ocean bottom. The method of obtaining soundings by means of the hand lead line is applicable to a depth of only about 20 fathoms. For greater depths, lead-line soundings may be made by means of mechanically operated sheaves, known as sounding machines. In either case, it is necessary for the survey boat to proceed at a slow rate, and in many cases to stop altogether, during the actual making of the soundings. This requirement renders the method of obtaining deep soundings by lead line rather slow and impracticable. Also, the distance that a coastal survey extends from the shore may be so great that the shore signals or the floating signals are no longer satisfactory for complete control of the positions of the soundings.

In order to facilitate the making of coastal hydrographic surveys in deep waters and at considerable distances from the shore, the U. S. Coast and Geodetic Survey has developed special methods for obtaining soundings while the survey boat is traveling at full speed and also for locating positions when signals are not visible from the survey boat. The two methods that have been widely used for obtaining soundings are (1) the *pressure-tube method*, and (2) the *echo-sounding method*. The positions of soundings made by either of these methods may be located by one of the two following methods: (1) *precise dead reckoning* and (2) *radio-acoustic sound ranging*. Where

echo sounding is used, the positions may also be determined by observations on stars; the entire procedure is usually called *star-control echo sounding.*

69. Pressure Tube.—The pressure-tube method of measuring the depth of water consists in submerging a hollow tube that is closed at the bottom end and is provided at the upper end with a special cap through which the water can enter but the air cannot escape. The amount of water that is forced into such a tube depends on the pressure of the water, and this pressure in turn depends on the depth to which the tube is lowered. Therefore, by measuring the amount of water in the tube, the depth to which the tube was lowered can be determined. Usually, two tubes are used so that one may be submerged while the amount of water in the other is being determined.

The pressure tube of the U. S. Coast and Geodetic Survey consists of a two-foot length of brass tubing, the inside diameter of which is ½ inch. One end of the tube is closed and the other is fitted with a special brass cap in which there is a small spiral-shaped capillary tube about 3 inches long. The sealed end of a pressure tube is weighted so that the tube will sink in a vertical position with the capped end uppermost. As the pressure tube is lowered, water is forced into it through the capillary opening, but no air can escape. The amount of water that is forced into the pressure tube depends on the pressure due to the depth of the water in which the tube is submerged. When the tube is raised out of the water, the air escapes and the water sinks to the bottom of the tube. After the tube has been brought on board the survey boat, the cap is removed and a special brass rod is inserted into the tube until the level of the contained water is forced up to the top of the tube. By means of a sliding marker on the brass rod and a specially-graduated scale that can be held against the rod, it is possible to read the depth, in fathoms, to which the pressure tube was lowered.

It is rather difficult to insert the brass rod just the right amount without spilling any of the water; and, in order to

avoid this difficulty, an electrically controlled indicator has been devised. This indicator is actuated by the brass rod that is inserted into the pressure tube. When the point of the rod strikes the water, a click is transmitted to a set of earphones in the electric circuit, and the reading of the indicator at that instant shows the depth, in fathoms, to which the pressure tube was lowered.

70. Echo Sounding.—The echo-sounding machine used by the U. S. Coast and Geodetic Survey is an electrically operated apparatus consisting of the following essential parts: An *oscillator*, which produces a sound under water; a *hydrophone*, or a device for detecting the echo of the sound as it is reflected from the bottom of the ocean below the point at which the sound was made; a *fathometer*, which is a device for measuring the interval of time between the production of the sound and the detection of its echo; a *filter*, or a device for amplifying the intensity of the echo and reducing the intensity of other under-water noises; a *motor generator*, which transforms the ship's electrical power to the proper voltage and the correct number of cycles needed for the oscillator; and the necessary wires and switches for connecting and operating the various units.

The method of echo sounding consists in producing a sound by means of an oscillator attached to the bottom of the survey boat, and obtaining an echo of the sound in a hydrophone which is also attached to the bottom of the boat. The time interval is recorded by a fathometer. However, the speed of sound in water varies between 4,740 feet and 5,220 feet per second, the exact speed depending on the temperature of the water, the pressure to which the water is subjected, and the salinity, or salt content, of the water. Usually, the depth is computed on the basis of a velocity of 4,800 feet per second, and this computed depth is multiplied by a correction factor in order to obtain the more correct depth. Correction factors for various temperatures, pressures, and salinities may be obtained from special tables that are published by the U. S. Coast and Geodetic Survey.

METHODS FOR LOCATING POSITIONS OF SOUNDINGS

71. Precise Dead Reckoning.—When the positions of soundings are to be determined by precise dead reckoning, the ship starts from a known point and runs along a course with a definite bearing for a certain distance to the point at which the first sounding is to be taken. The starting point may be a floating signal or a point located by measuring sextant angles to three floating signals. The speed of the ship is maintained as nearly constant as is possible so that the distance the ship travels can be estimated from the elapsed time; also, the course is set so as to allow for the effect of wind and current. From the first sounding, the ship is run along the proper course and for the required distance to the second sounding; and each successive sounding is located similarly from the preceding sounding. When the limiting distance from shore has been reached, the ship runs parallel to the shore for a definite distance and proceeds back toward shore on a calculated course which is parallel to the outgoing course. Finally, the position of the ship at the shore end of the trip is checked by either finishing at a floating signal or sighting on floating signals. In order to make allowance for the wind and the current, it is customary to measure the wind velocity and the set and drift of the current at the start and finish of the trip and at intervals of 2 hours during the trip.

The use of this method is confined largely to localities where the ocean bottom is comparatively level and the soundings are required only at points that are a considerable distance apart. Usually, for depths up to 15 fathoms the soundings are made about 1 to 2 miles apart; and for depths from 15 to 100 fathoms the soundings are made about 2 to 4 miles apart. Although dead reckoning is often used for locating soundings where floating signals cannot be placed economically for complete control, the use of this method is limited by the following three factors: (1) The necessity for obtaining the closing error of the courses followed by the survey boat, before the probable position of the soundings can be definitely determined; (2) the difficulty encountered in covering a definite distance at a pre-

determined speed; and (3) the probable inaccuracies in the calculated positions that may be caused by variations in the speed of the boat.

72. Radio-Acoustic Sound Ranging.—In radio-acoustic sound ranging, the soundings are made along ranges whose control points are located as follows: A bomb is exploded in the water at each control point, the exact time of the explosion is determined by observations made on the ship, and the exact

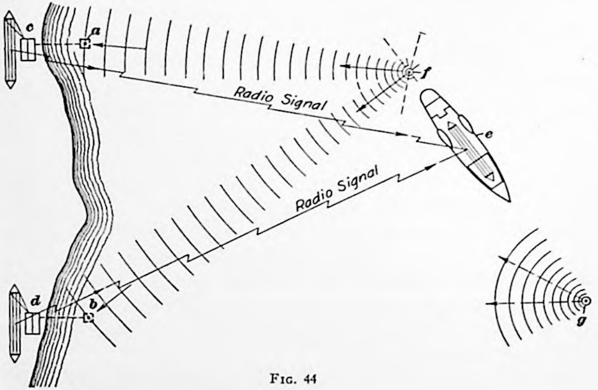

Fig. 44

time at which the sound of the explosion reaches each of two established hydrophone stations is recorded at the ship by means of a radio signal. The distance between the two hydrophone stations is known, and the distance from the control point to each of the two stations can be computed by multiplying the velocity of sound in water by the time it took for the sound to travel from the point of explosion to the respective station.

In Fig. 44 is shown diagrammatically the procedure for locating by radio-acoustic sound ranging the position of a ship that is making off-shore soundings. Two hydrophone stations

a and *b* are placed about 4 or 5 feet from the ocean bottom in 7 to 10 fathoms of water. The positions of these hydrophone stations are accurately located by sextant angles to known stations on shore. Also, each hydrophone station is connected by a cable to the transmitter of a temporary radio station, as *a* to *c* and *b* to *d*. In the bottom of the ship *e* is a hydrophone that is connected to an electrically operated recording tape, or *chronograph*. The ship is also equipped with a radio receiving set that is connected to the chronograph.

The position of the ship is determined as follows: A small bomb, which is equipped with a fuse that will cause it to explode 20 to 30 seconds after it strikes the water, is dropped overboard while the ship is traveling at a predetermined speed. The bomb will normally sink at the rate of about a fathom per second. In shallow waters, where the bomb would reach the bottom before it explodes, the bomb may be held at a certain depth by attaching it by means of a cord to a temporary buoy consisting of a paper bag filled with air. At the instant the bomb is dropped, an electrically controlled relay marks the tape on the chronograph. Also, when the bomb explodes, as indicated at *f*, the sound of the explosion is received by the hydrophone on the ship and is relayed to the chronograph. From the speed of the ship, the speed of sound in water, and the total elapsed time between the dropping of the bomb and the receipt of the explosion, the exact time of the explosion may be computed.

The sound of the explosion of the bomb at *f* also travels under water to the hydrophone stations *a* and *b*. At the instant the sound impulse is received at *a*, a signal is sent by cable to the radio station at *c*, from which it is transmitted to the ship. Likewise, at the instant the sound impulse is received at *b*, a signal is transmitted to the radio station at *d* and to the ship. As each radio signal is received by the ship's set, the chronograph records the exact time of its receipt. From these times, the calculated exact time of the explosion, and the velocity of sound in water, it is possible to compute the distances *af* and *bf*, and thus to locate point *f* accurately. If another bomb is dropped when the ship reaches point *g*, the

distances *ag* and *bg* can also be determined in the manner explained for *af* and *bf*. From the known positions of *f* and *g* and the known speed of the ship, it is possible to locate any number of soundings that may be made between points *f* and *g*. Usually, the velocity of sound in the water in the vicinity of the work is determined by tests made while the ship is near the shore, and the necessary angles to the signals can be measured with a sextant.

EXAMPLE.—Determine the distances *af* and *bf* in Fig. 44 under the following conditions: The elapsed times between the explosion of the bomb at *f* and the receipt of the signals from the radio stations at *c* and *d* are 97.84 seconds and 88.60 seconds, respectively; and the average velocity of sound in the water in the vicinity of the work is found from tests to be 4,859 feet per second.

SOLUTION.—Each of the required distances is found by multiplying the velocity of sound by the time interval between the explosion of the bomb and the receipt of the radio signal at the ship. Thus,

$$af = 97.84 \times 4{,}859 = 475{,}405 \text{ ft.} \quad \text{Ans.}$$

and

$$bf = 88.60 \times 4{,}859 = 430{,}507 \text{ ft.} \quad \text{Ans.}$$

73. Star-Control Echo Sounding.—The method of star control for determining the positions of soundings is particularly adaptable to soundings made by means of echo-sounding machines on ranges that extend a considerable distance from the shore. The ship is started in a calculated direction, as obtained by precise dead reckoning, from a known signal on or near the shore. When the ship reaches the end of the run, its position is determined by observations on several stars with a sextant. The observations should preferably be made on four stars that are about equally spaced around the horizon. In order to make satisfactory observations, the horizon and the stars should both be clearly visible. These conditions require an exceptionally clear sky and limit the time for making the observations to either dawn or dusk. After the star observations have been made, the ship is run back toward shore on a different course from the one for the outgoing trip and the final position of the ship is checked by observations on known stations on or near the shore.

During both the outgoing and ingoing runs, the speed of the ship should be kept as nearly constant as possible so that the distances between soundings may be calculated accurately from the time intervals between soundings. Occasionally, during a run, observations may be taken on the sun or the moon for the purpose of checking the position of the ship.

The difficulty encountered in this method is chiefly that of keeping the ship on its course and maintaining a constant speed. Also, conditions for making star observations at the end of an outgoing run are not always satisfactory, as the sky may become cloudy between the starting time and the time of completion of the run. Then the entire work must be discarded, and the run must be started again.